Continuous Probability Distributions

Chapter 07

LEARNING OBJECTIVES

LO 7-1 List the characteristics of the uniform distribution.

LO 7-2 Compute probabilities by using the uniform distribution.

LO 7-3 List the characteristics of the normal probability distribution.

LO 7-4 Convert a normal distribution to the standard normal distribution.

LO 7-5 Find the probability that an observation on a normally distributed random variable is between two values.

LO 7-6 Find probabilities using the Empirical Rule.

LO 7-1 List the characteristics of the uniform distribution.

The Uniform Distribution

The uniform probability distribution is perhaps the **simplest distribution for a continuous random variable**.

This distribution is **rectangular in shape** and is defined by minimum and maximum values.

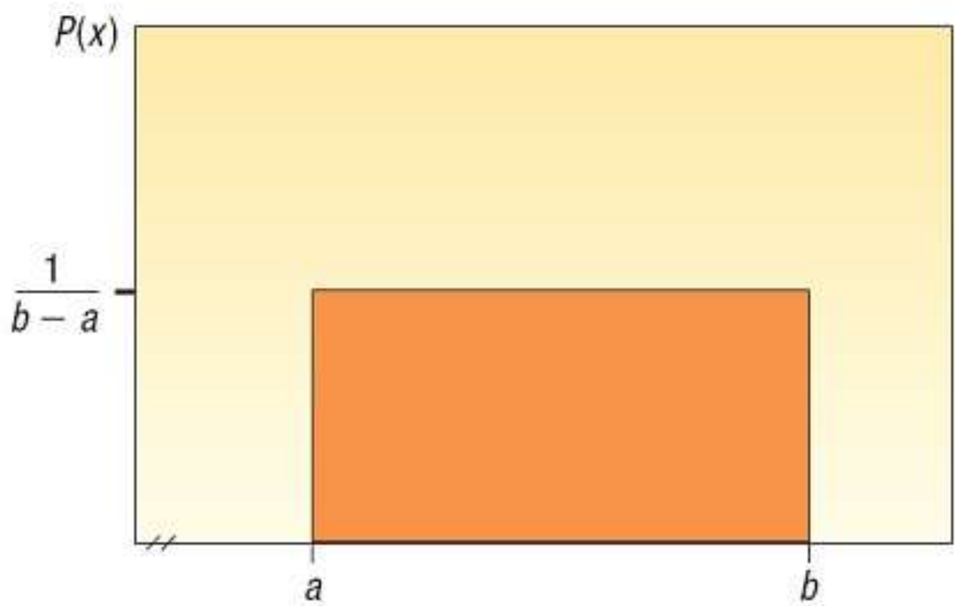


CHART 7–1 A Continuous Uniform Distribution

The Uniform Distribution – Mean and Standard Deviation

MEAN OF THE UNIFORM DISTRIBUTION $$\mu = \frac{a+b}{2}$$ **[7–1]**

STANDARD DEVIATION OF THE UNIFORM DISTRIBUTION $$\sigma = \sqrt{\frac{(b-a)^2}{12}}$$ **[7–2]**

UNIFORM DISTRIBUTION $$P(x) = \frac{1}{b-a} \quad \text{if } a \leq x \leq b \text{ and 0 elsewhere}$$ **[7–3]**

LO 7-2 Compute probabilities using the uniform distribution.

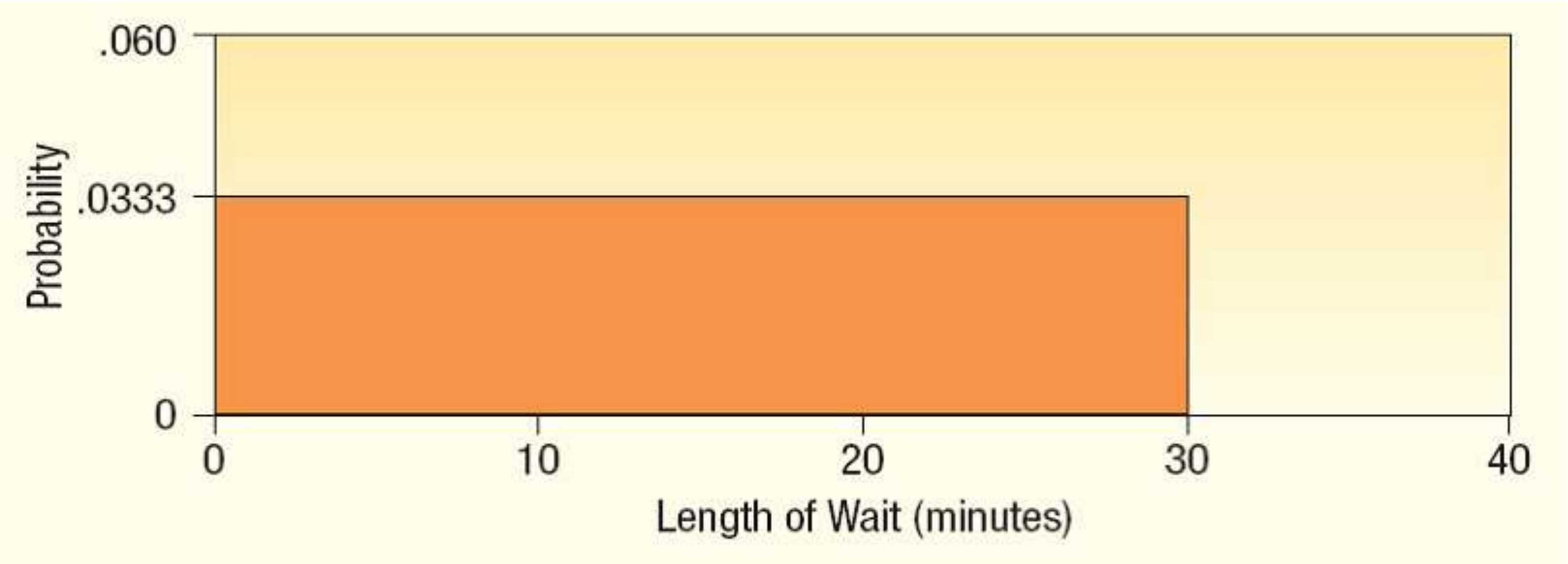
The Uniform Distribution – Example

Southwest Arizona State University provides bus service to students while they are on campus. A bus arrives at the North Main Street and College Drive stop every 30 minutes between 6 A.M. and 11 P.M. during weekdays. Students arrive at the bus stop at random times. The time that a student waits is uniformly distributed from 0 to 30 minutes.

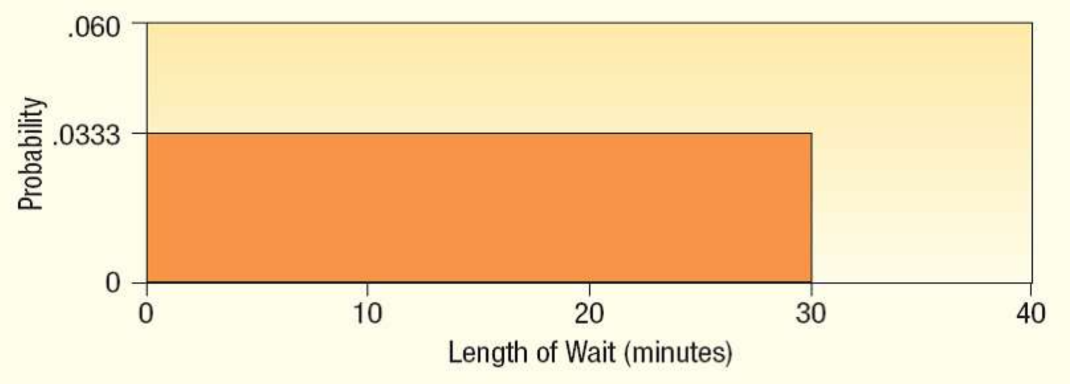
1. Draw a graph of this distribution.
2. Show that the area of this uniform distribution is 1.00.
3. How long will a student "typically" have to wait for a bus? In other words, what is the mean waiting time? What is the standard deviation of the waiting times?
4. What is the probability a student will wait more than 25 minutes?
5. What is the probability a student will wait between 10 and 20 minutes?

The Uniform Distribution – Example

1. Graph of this distribution.



The Uniform Distribution – Example

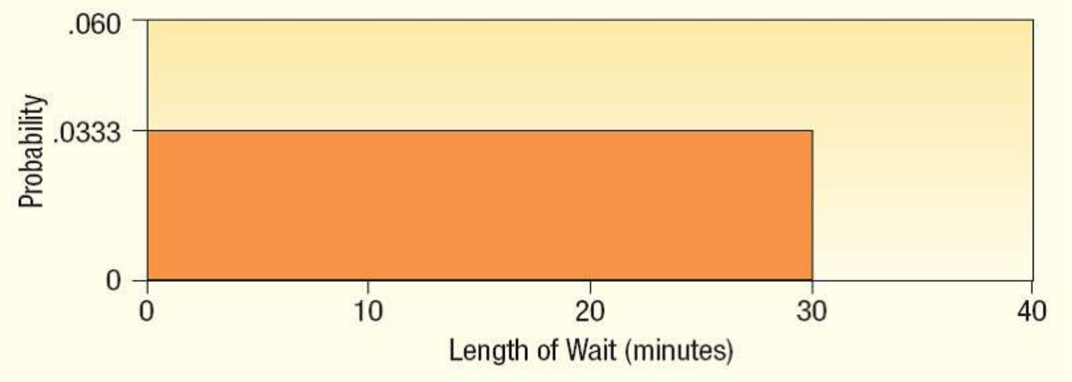


2. Show that the area of this distribution is 1.00.

The times students must wait for the bus is uniform over the interval from 0 minutes to 30 minutes, so in this case *a* is 0 and *b* is 30.

$$\text{Area} = (\text{height})(\text{base}) = \frac{1}{(30 - 0)}(30 - 0) = 1.00$$

The Uniform Distribution – Example



3. How long will a student "typically" have to wait for a bus? In other words, what is the **mean waiting time**?

$$\mu = \frac{a+b}{2} = \frac{0+30}{2} = 15$$

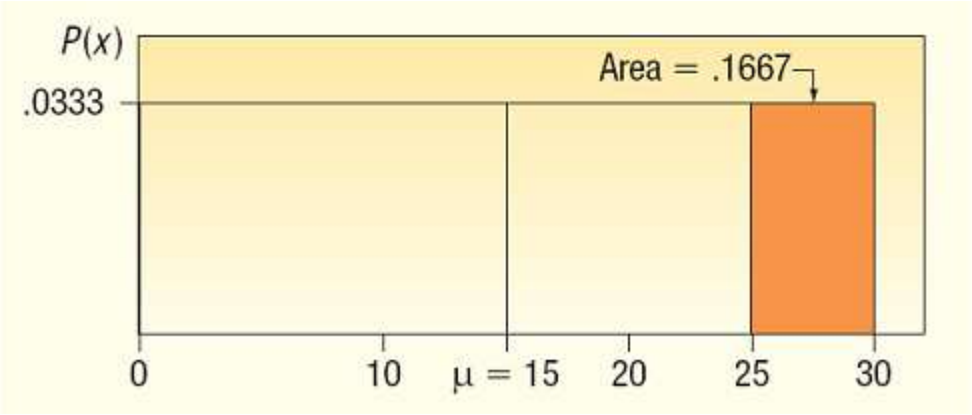
What is the **standard deviation** of the waiting times?

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(30-0)^2}{12}} = 8.66$$

The Uniform Distribution – Example

4. What is the probability a student will wait **more than 25 minutes**?

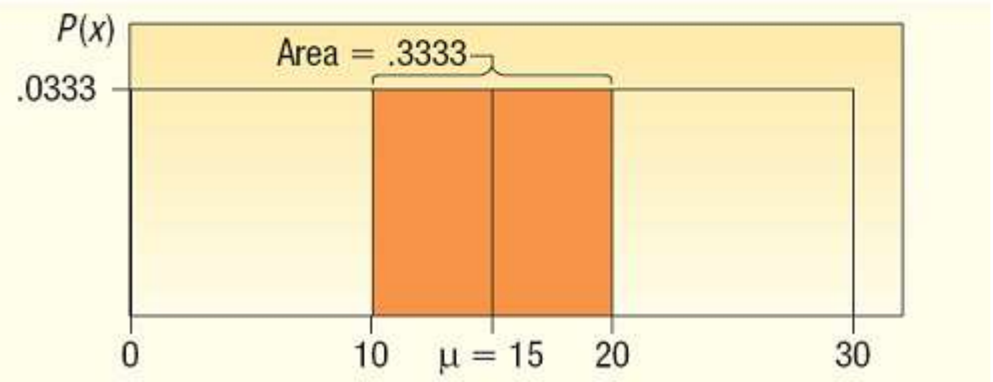
$$P(25 < \text{Wait Time} < 30) = (\text{height})(\text{base})$$
$$= \frac{1}{(30-0)}(5)$$
$$= 0.1667$$



The Uniform Distribution – Example

5. What is the probability a student will wait **between 10 and 20** minutes?

$$P(10 < \text{Wait Time} < 20) = (\text{height})(\text{base})$$
$$= \frac{1}{(30-0)}(10)$$
$$= 0.3333$$

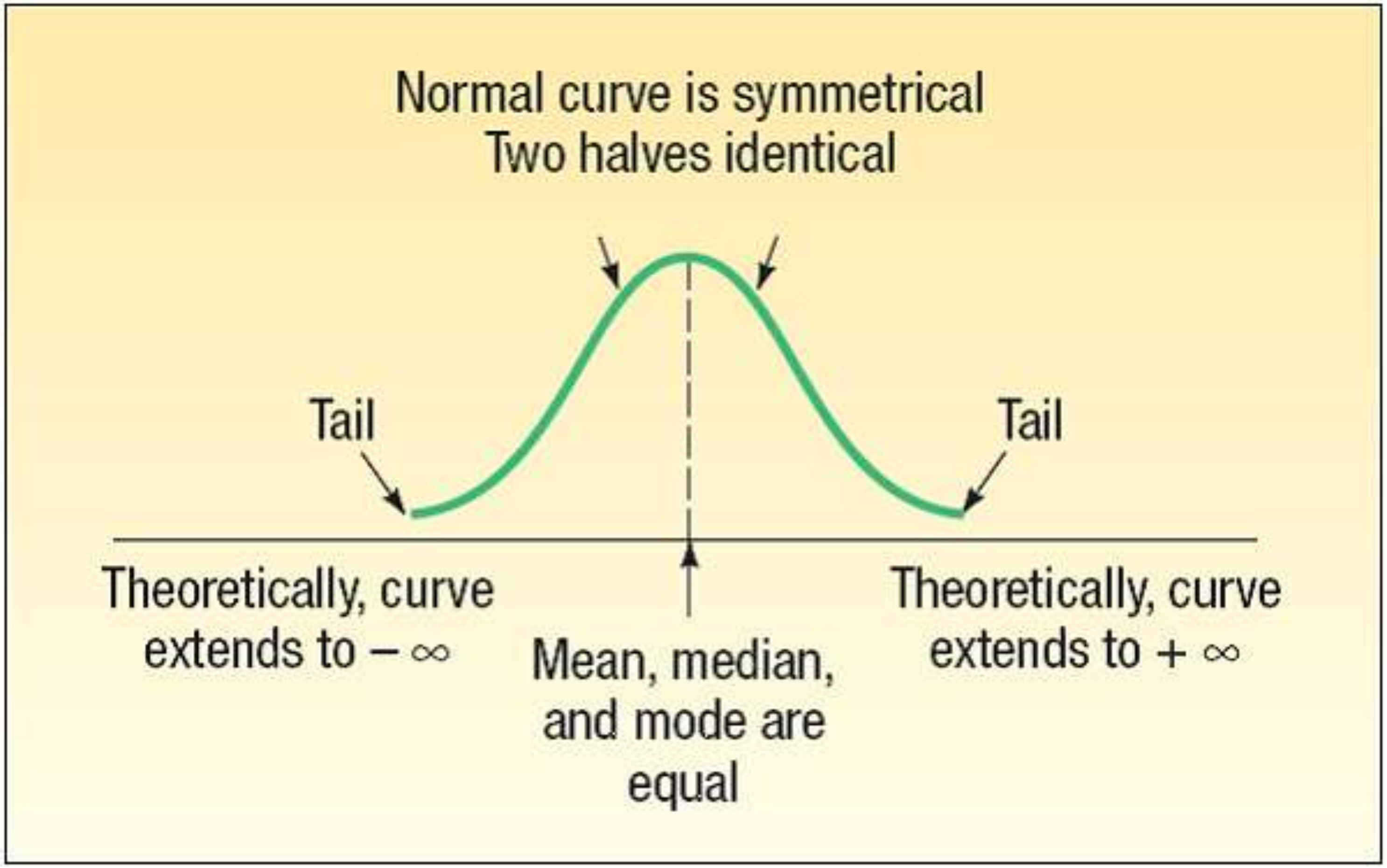


LO 7-3 List the characteristics of the normal probability distribution.

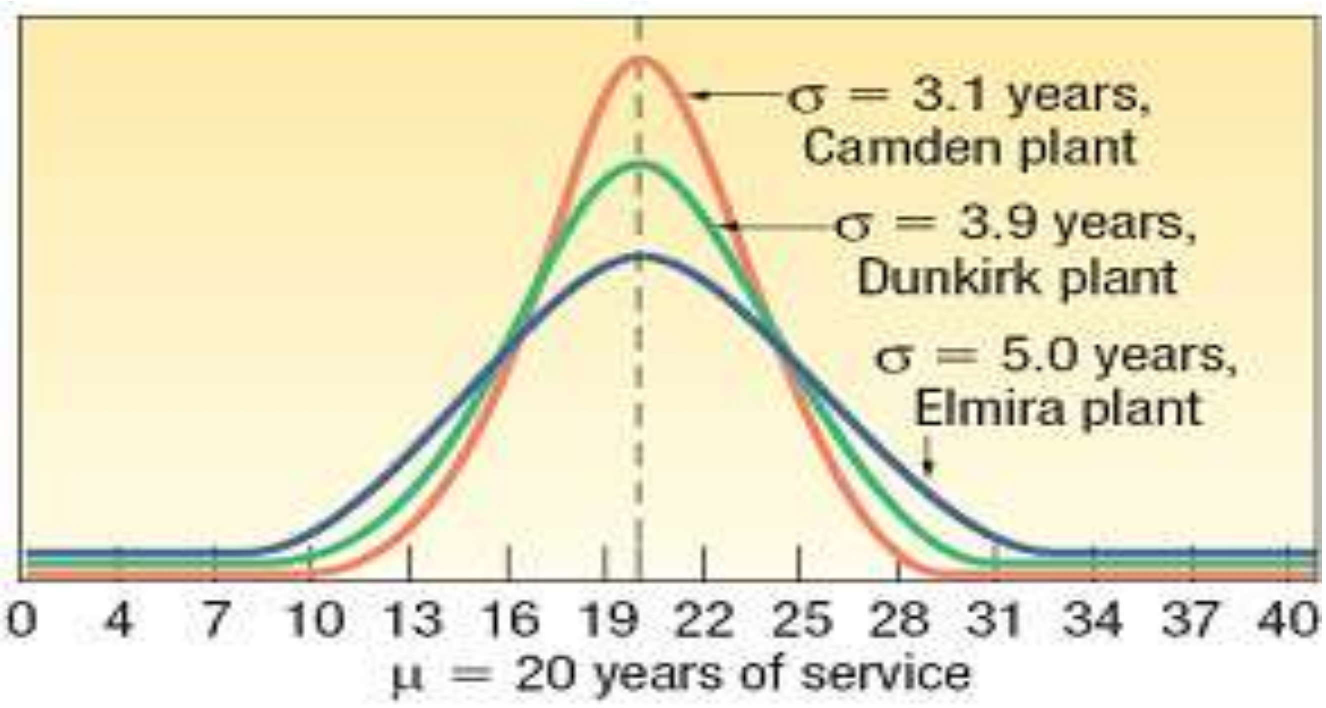
Characteristics of a Normal Probability Distribution

1. It is **bell-shaped** and has a single peak.
2. It is **symmetrical** about the mean.
3. It is **asymptotic**: The curve gets closer and closer to the *X*-axis but never actually touches it.
4. The arithmetic **mean, median**, and **mode are equal**
5. The total **area under the curve is 1.00**.
6. The area to the left of the mean = area right of mean = 0.5.

The Normal Distribution – Graphically

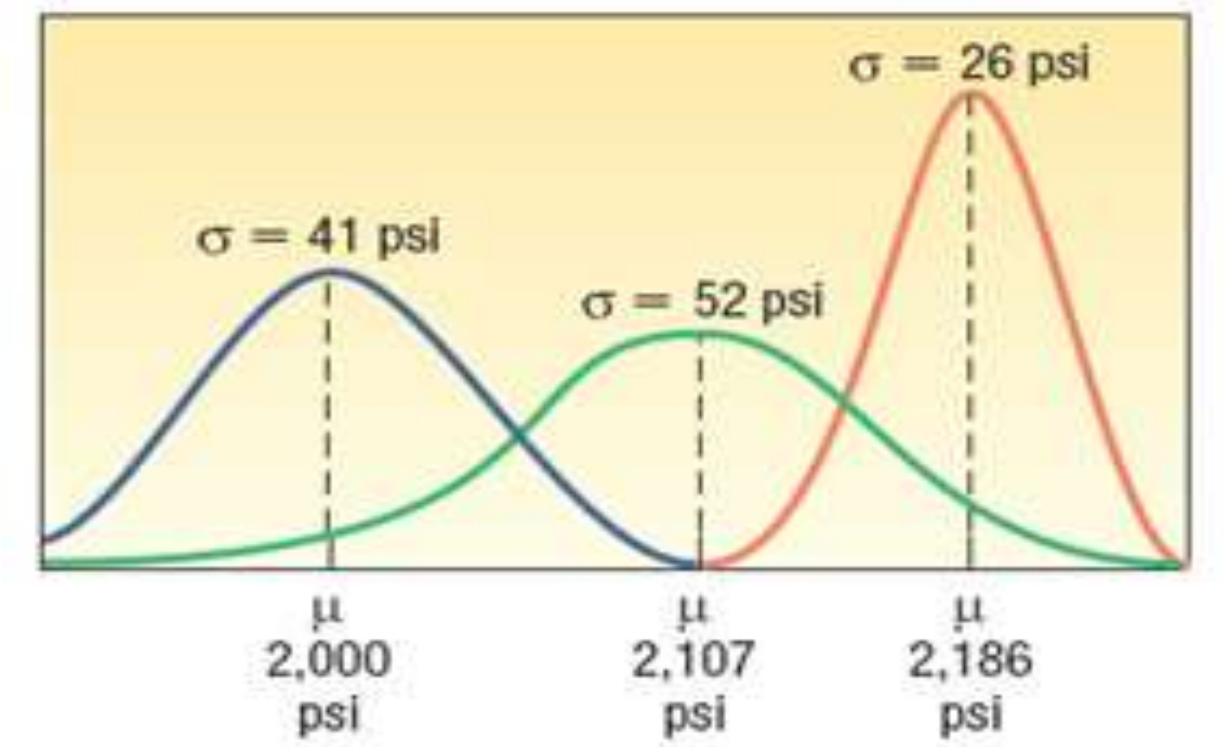


The Family of Normal Distribution



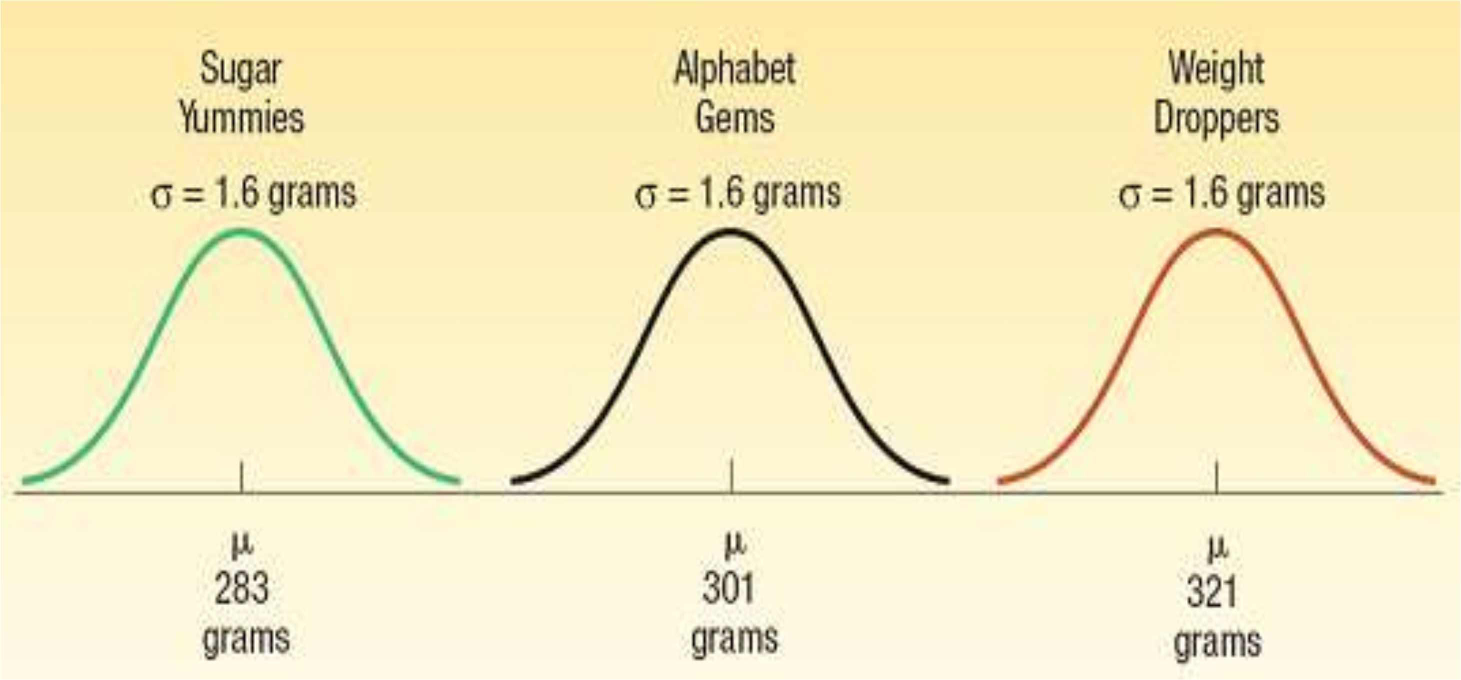
Equal Means and Different Standard Deviations

The Family of Normal Distribution



Different Means and Standard Deviations

The Family of Normal Distribution



Different Means and Equal Standard Deviations

LO 7-4 Convert a normal distribution to the standard normal distribution.

The Standard Normal Probability Distribution

- The standard normal distribution is a normal distribution with a **mean of 0** and a **standard deviation of 1**.
- It is also called the ***z* distribution**.
- A ***z*-value** is the signed distance between a selected value, designated *X*, and the population mean μ, divided by the population standard deviation, σ.
- The formula is:

$$z = \frac{X - \mu}{\sigma}$$

Areas Under the Normal Curve

z	0.00	0.01	0.02	0.03	0.04	0.05	. . .
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	
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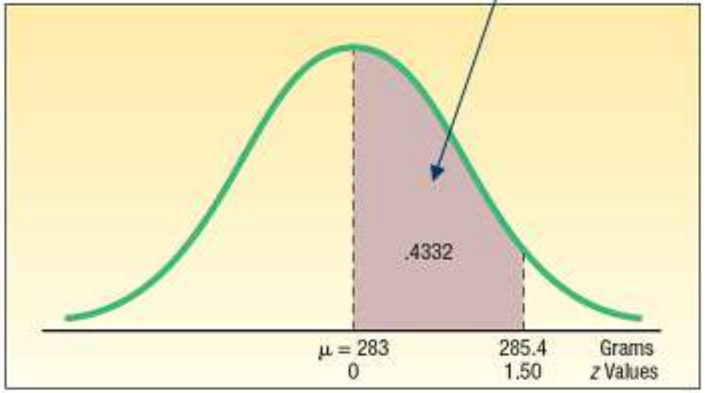


LO 7-5 Find the probability that an observation on a normally distributed random variable is between two values.

The Normal Distribution – Example

The weekly incomes of shift foremen in the glass industry follow the normal probability distribution with a **mean of $1,000** and a **standard deviation of $100**.

What is the ***z*-value** for the income, let's call it *X*, of a foreman who earns **$1,100** per week? For a foreman who earns **$900** per week?

$$\text{For X} = \$1,100:$$

$$z = \frac{X - \mu}{\sigma} = \frac{\$1,100 - \$1,000}{\$100} = 1.00$$

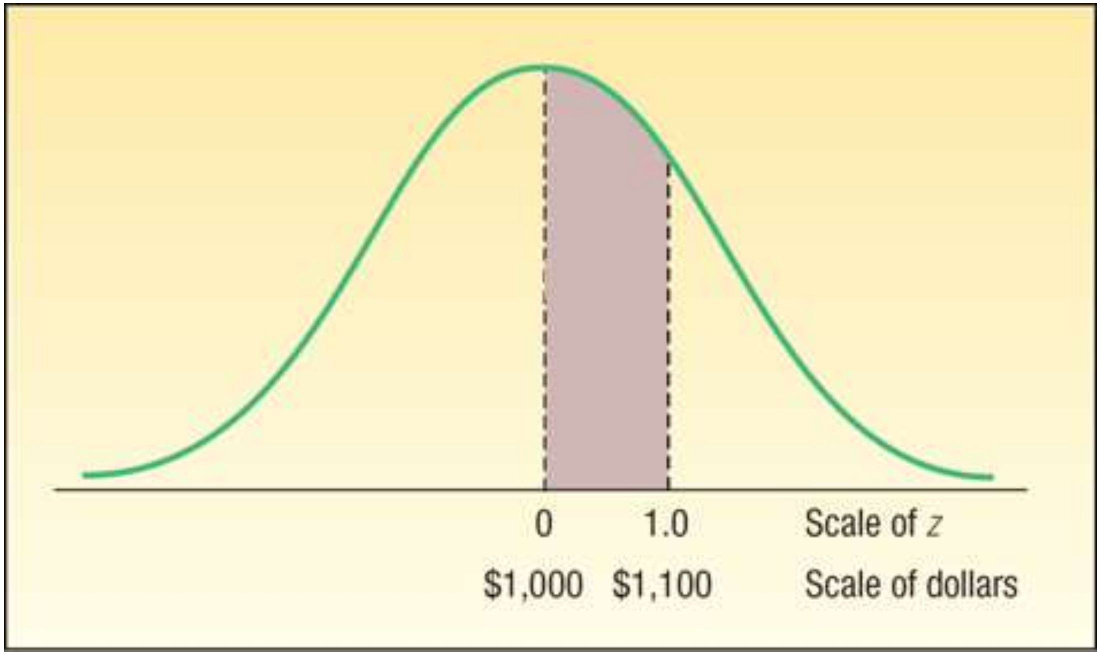
$$\text{For X} = \$900:$$

$$z = \frac{X - \mu}{\sigma} = \frac{\$900 - \$1,000}{\$100} = -1.00$$

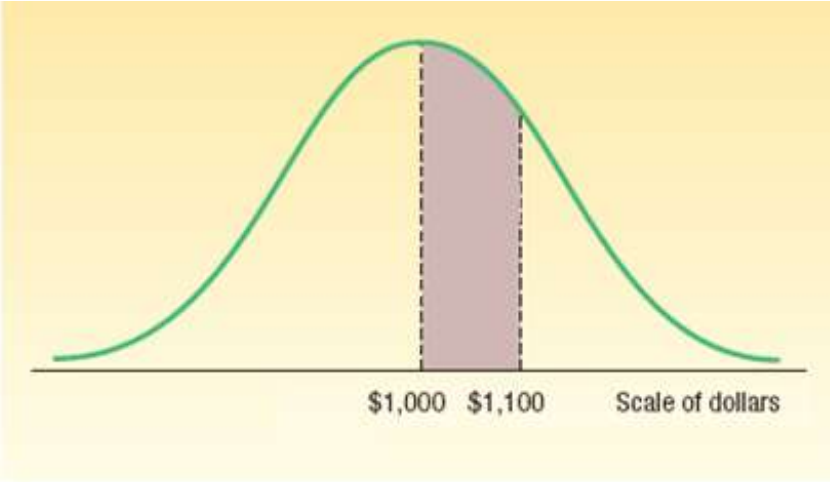
Normal Distribution – Finding Probabilities

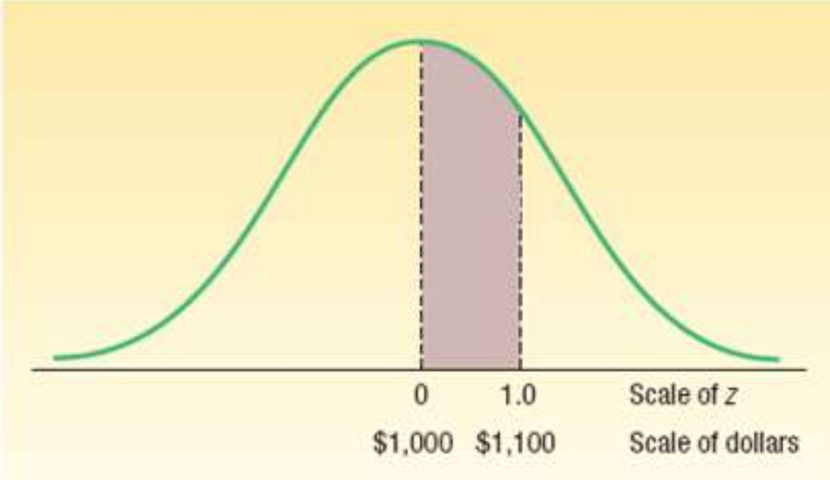
In an earlier example, we reported that the mean weekly income of a shift foreman in the glass industry is normally distributed with a mean of $1,000 and a standard deviation of $100.

What is the likelihood of selecting a foreman whose weekly income is between $1,000 and $1,100?



Normal Distribution – Finding Probabilities





For X = $1,000:

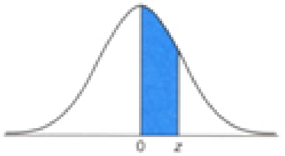
$$z = \frac{X - \mu}{\sigma} = \frac{\$1{,}000 - \$1{,}000}{\$100} = 0.00$$

For X = $1,100:

$$z = \frac{X - \mu}{\sigma} = \frac{\$1{,}100 - \$1{,}000}{\$100} = 1.00$$

Normal Distribution – Finding Probabilities Using the Normal Distribution Table

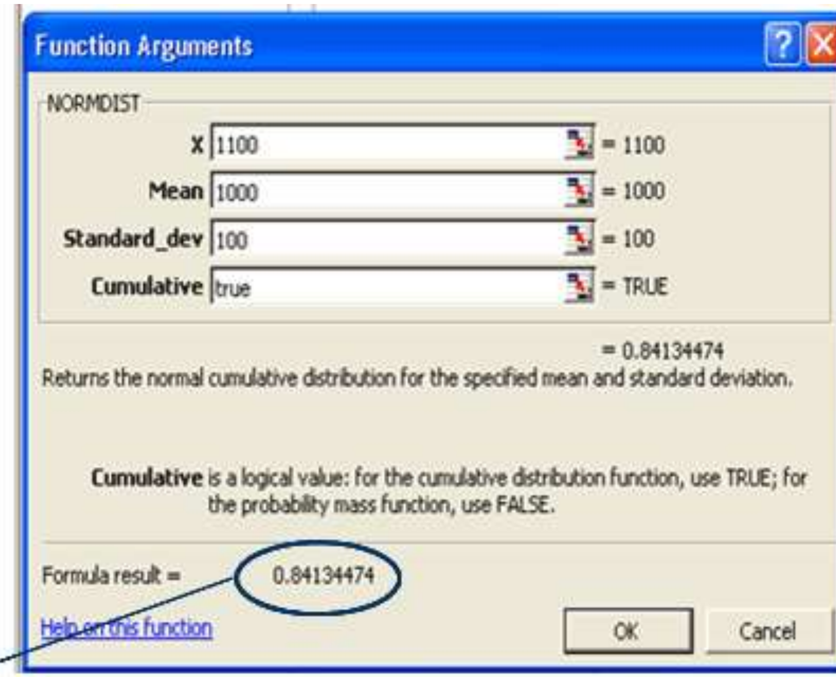
Areas under the normal curve, 0 to z

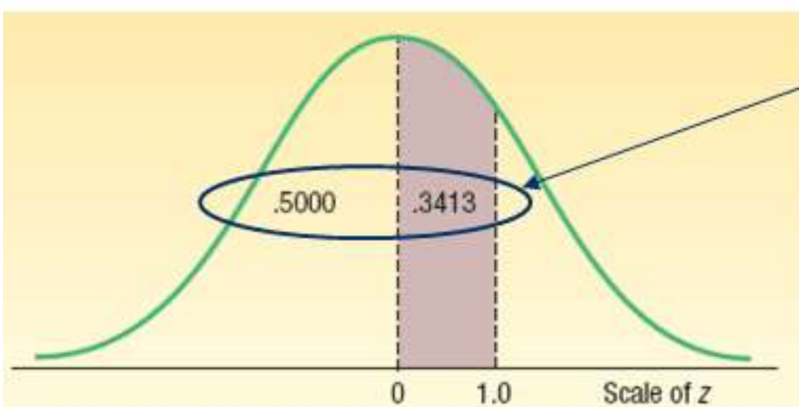


z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699

Finding Areas for *z* Using Excel

The Excel function
=NORMDIST(x,Mean,Standard_dev,Cumu)
=NORMDIST(1100,1000,100,true)
generates area (probability) from
Z=1 and below





Normal Distribution – Finding Probabilities (Example 2)

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution with a mean of \$1,000 and a standard deviation of \$100.

What is the probability of selecting a shift foreman in the glass industry whose income is:

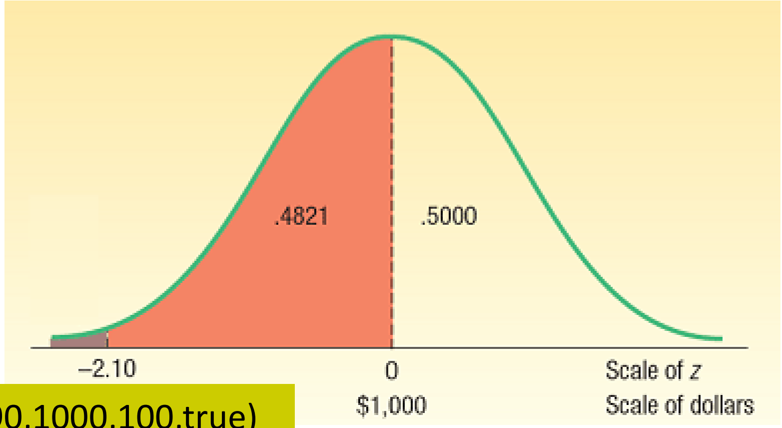
Between \$790 and \$1,000?

For X = \$790:

$$z = \frac{X - \mu}{\sigma} = \frac{\$790 - \$1{,}000}{\$100} = -2.10$$

For X = \$1,000:

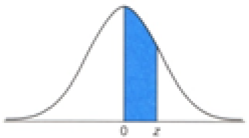
$$z = \frac{X - \mu}{\sigma} = \frac{\$1{,}000 - \$1{,}000}{\$100} = 0.00$$



Excel Function: =NORMDIST(1000,1000,100,true)-NORMDIST(790,1000,100,true)

Normal Distribution – Finding Probabilities using the Normal Distribution Table

Areas under the normal curve, 0 to z



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714
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0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162
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1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887

Normal Distribution – Finding Probabilities (Example 3)

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the probability of selecting a shift foreman in the glass industry whose income is:

Less than $790?

Find Z for X = $790 :

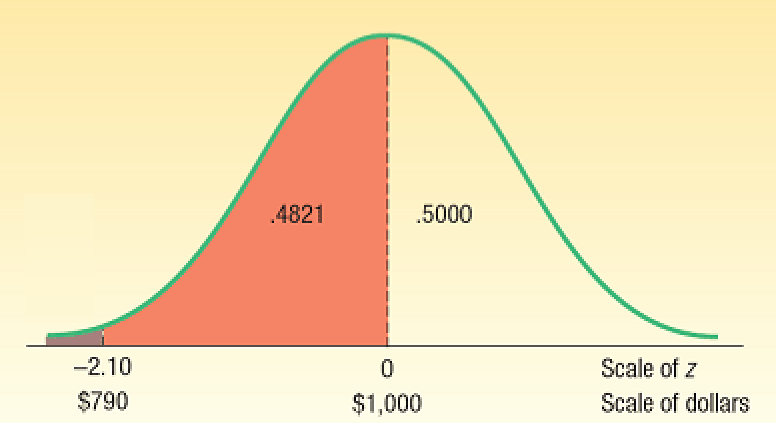
$$z = \frac{X-\mu}{\sigma} = \frac{\$790-\$1{,}000}{\$100} = -2.10$$

To find the are below - 2.10,

subtract from 0.50 the area from - 2.10 to 0

= 0.50-0.4821

= 0.0179

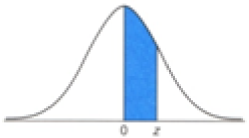


Excel Function: =NORMDIST(790,1000,100,true)

LO 7-5

Normal Distribution – Finding Probabilities Using the Normal Distribution Table

Areas under the normal curve, 0 to z



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162
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1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887

Normal Distribution – Finding Probabilities (Example 4)

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the probability of selecting a shift foreman in the glass industry whose income is:

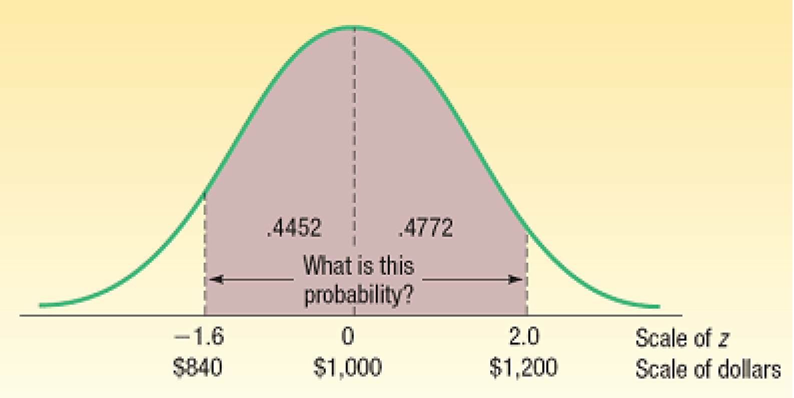
Between $840 and $1,200?

For $X = \$840$:

$$z = \frac{X-\mu}{\sigma} = \frac{\$840-\$1,000}{\$100} = -1.60$$

For $X = \$1,200$:

$$z = \frac{X-\mu}{\sigma} = \frac{\$1,200-\$1,000}{\$100} = 2.00$$

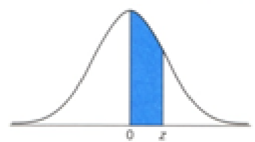


Excel Function: =NORMSDIST(2.0)-NORMSDIST(-1.6)

LO 7-5

Normal Distribution – Finding Probabilities Using the Normal Distribution Table

Areas under the normal curve, 0 to z



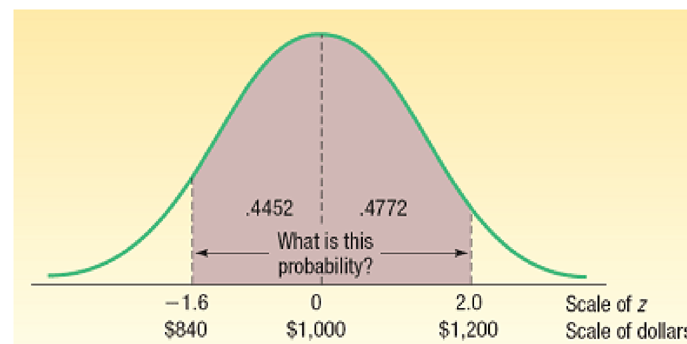
z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887

For X = \$840:

$$z = \frac{X-\mu}{\sigma} = \frac{\$840-\$1{,}000}{\$100} = -1.60$$

For X = \$1,200:

$$z = \frac{X-\mu}{\sigma} = \frac{\$1{,}200-\$1{,}000}{\$100} = 2.00$$



LO 7-5

Normal Distribution – Finding Probabilities (Example 5)

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the probability of selecting a shift foreman in the glass industry whose income is:

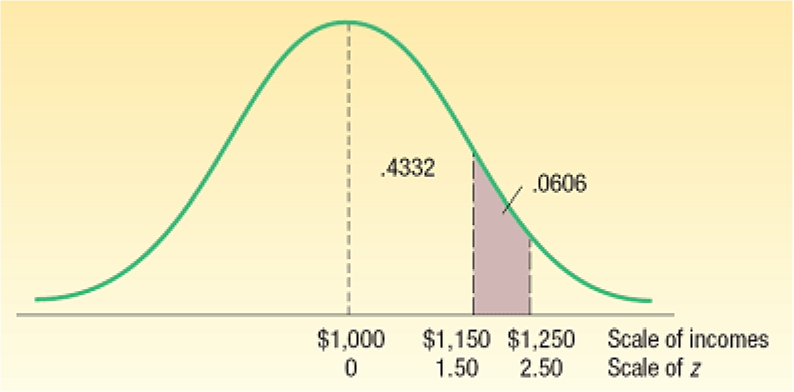
Between $1,150 and $1,250

For $X = \$1,150$:

$$z = \frac{X - \mu}{\sigma} = \frac{\$1,150 - \$1,000}{\$100} = 1.50$$

For $X = \$1,250$:

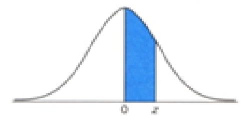
$$z = \frac{X - \mu}{\sigma} = \frac{\$1,250 - \$1,000}{\$100} = 2.50$$



Excel Function: =NORMSDIST(2.5)-NORMSDIST(1.5)

Normal Distribution – Finding Probabilities Using the Normal Distribution Table

Areas under the normal curve, 0 to z



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
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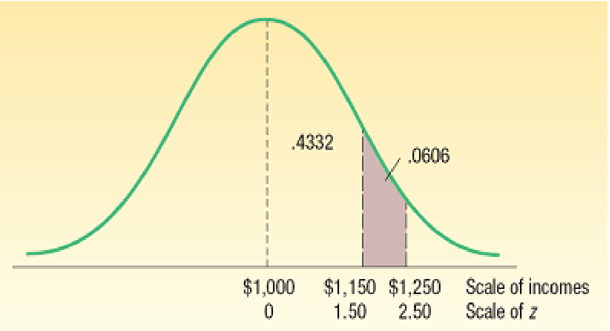
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973

For X = $1,150:

$$z = \frac{X - \mu}{\sigma} = \frac{\$1{,}150 - \$1{,}000}{\$100} = 1.50$$

For X = $1,250:

$$z = \frac{X - \mu}{\sigma} = \frac{\$1{,}250 - \$1{,}000}{\$100} = 2.50$$



Using *z* in Finding X Given Area – Example

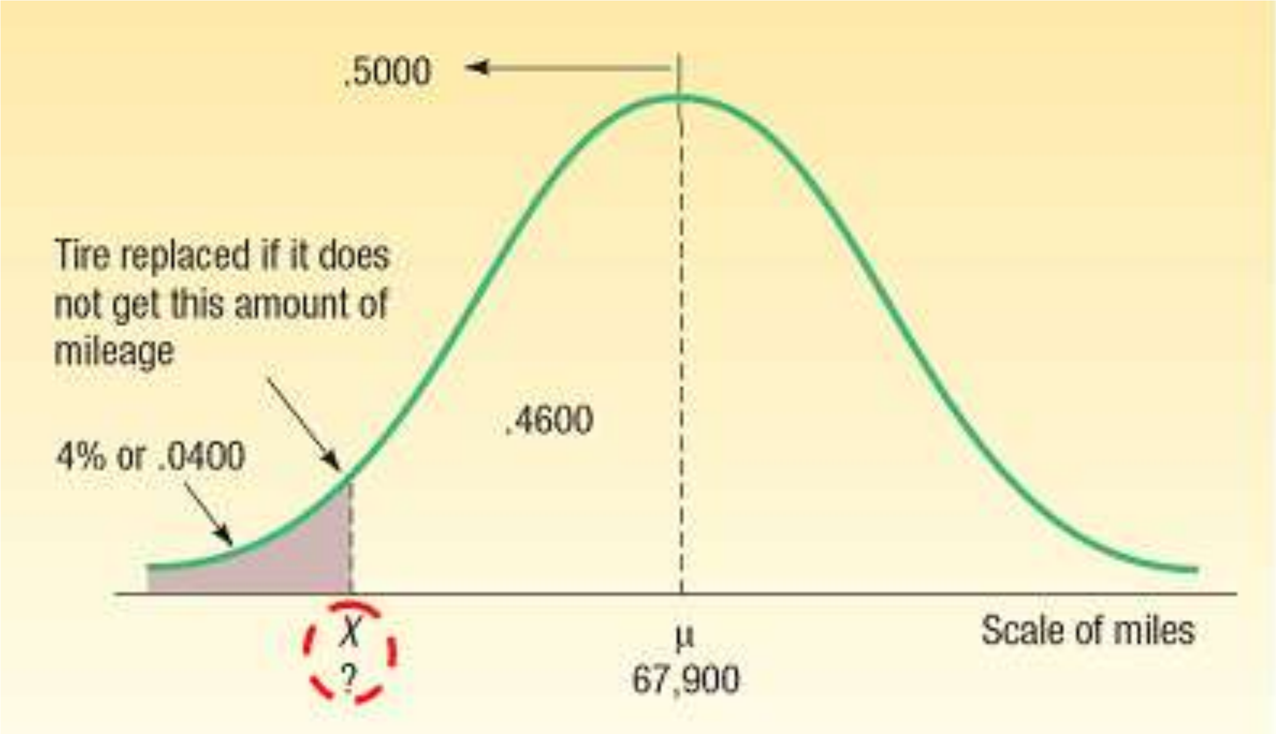
Layton Tire and Rubber Company wishes to set a minimum mileage guarantee on its new MX100 tire. Tests reveal the mean mileage is 67,900 with a standard deviation of 2,050 miles and that the distribution of miles follows the normal probability distribution. Layton wants to set the minimum guaranteed mileage so that no more than 4 percent of the tires will have to be replaced.

What minimum guaranteed mileage should Layton announce?

Using z in Finding X Given Area – Example

Set the minimum guaranteed mileage (X) so that no more than 4 percent of the tires will be replaced.

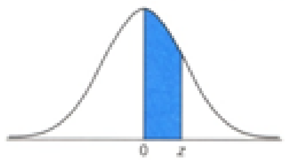
Given Data:
μ = 67,900
σ = 2,050
X = ?



Using z in Finding X Given Area – Example

Solve X using the formula :

$$z = \frac{x - \mu}{\sigma} = \frac{x - 67{,}900}{2{,}050}$$



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0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319
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1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699

Using z in Finding X Given Area – Example

Solve X using the formula :

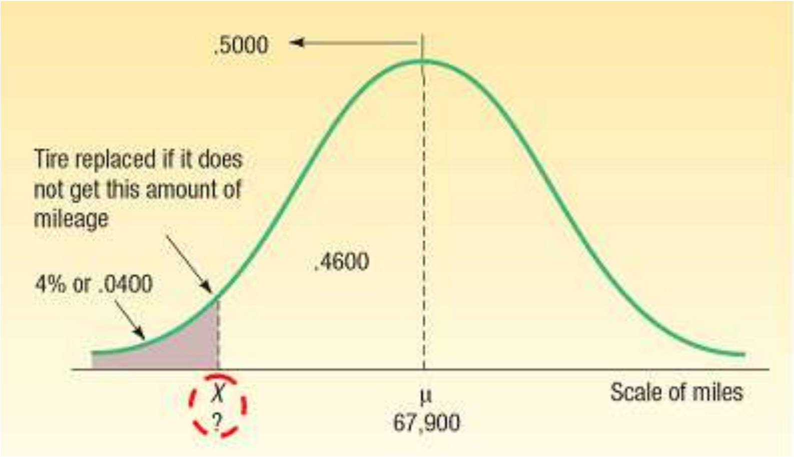
$$z = \frac{x - \mu}{\sigma} = \frac{x - 67{,}900}{2{,}050}$$

$$-1.75 = \frac{x - 67{,}900}{2{,}050}\text{, then solving for x}$$

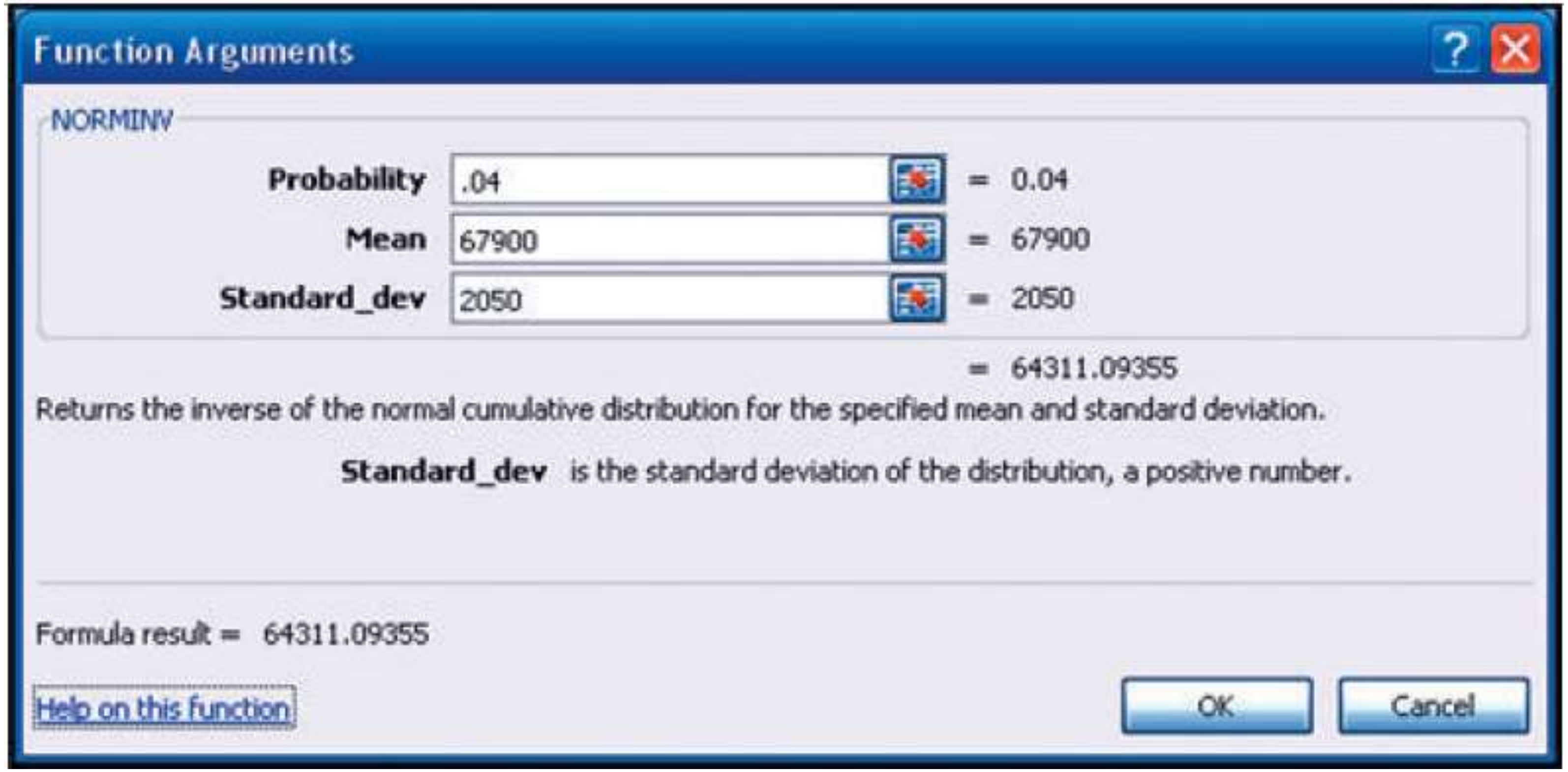
$$-1.75(2{,}050) = x - 67{,}900$$

$$x = 67{,}900 - 1.75(2{,}050)$$

$$x = 64{,}312$$



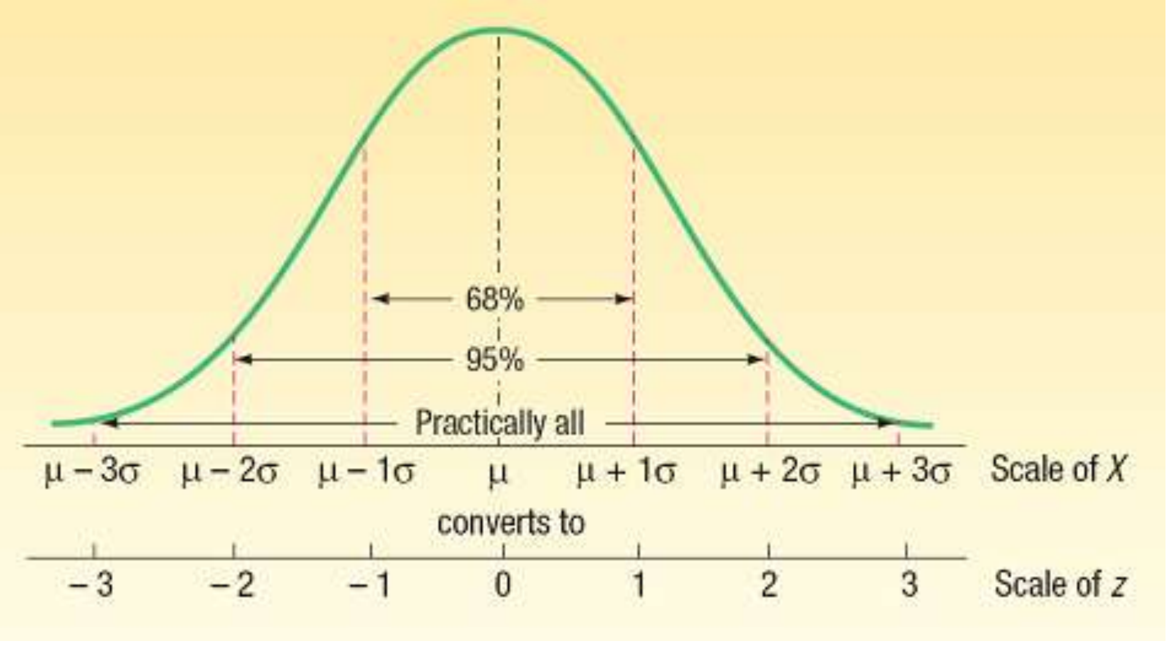
Using z in Finding X Given Area – Excel



LO 7-6 Find probabilities using the Empirical Rule.

The Empirical Rule

- About **68 percent** of the area under the normal curve is within **one standard deviation** of the mean.
- About **95 percent** is within **two standard deviations** of the mean.
- **Practically all** is within **three standard deviations** of the mean.



The Empirical Rule – Example

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours.

Answer the following questions.

1. About 68 percent of the batteries failed between what two values?
2. About 95 percent of the batteries failed between what two values?
3. Virtually all of the batteries failed between what two values?

The Empirical Rule – Example

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours.

We can use the results of the Empirical Rule to answer these questions.

1. About 68 percent of the batteries will fail between 17.8 and 20.2 hours by $19.0 \pm 1(1.2)$ hours.
2. About 95 percent of the batteries will fail between 16.6 and 21.4 hours by $19.0 \pm 2(1.2)$ hours.
3. Virtually all failed between 15.4 and 22.6 hours, found by $19.0 \pm 3(1.2)$

This information is summarized on the following chart.