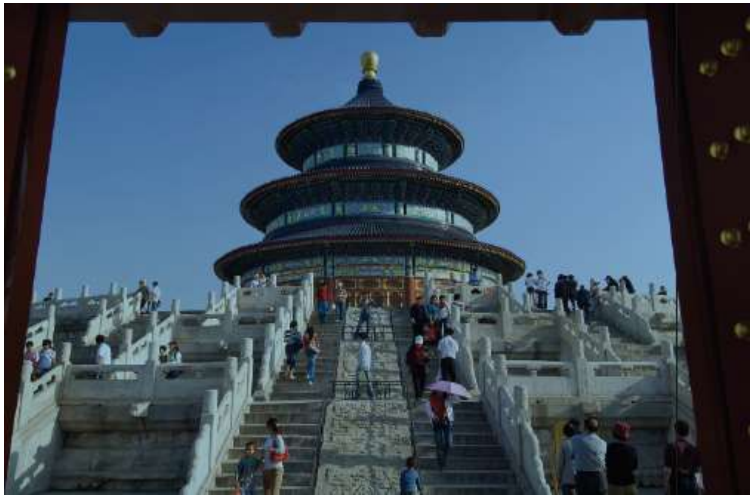
A Survey of Probability Concepts

Chapter 05



LEARNING OBJECTIVES

LO 5-1 Explain the terms *experiment, event, and outcome.*

LO 5-2 Identify and apply the appropriate approach to assigning probabilities.

LO 5-3 Calculate probabilities using the *rules of addition.*

LO 5-4 Define the term *joint probability.*

LO 5-5 Calculate probabilities using the *rules of multiplication.*

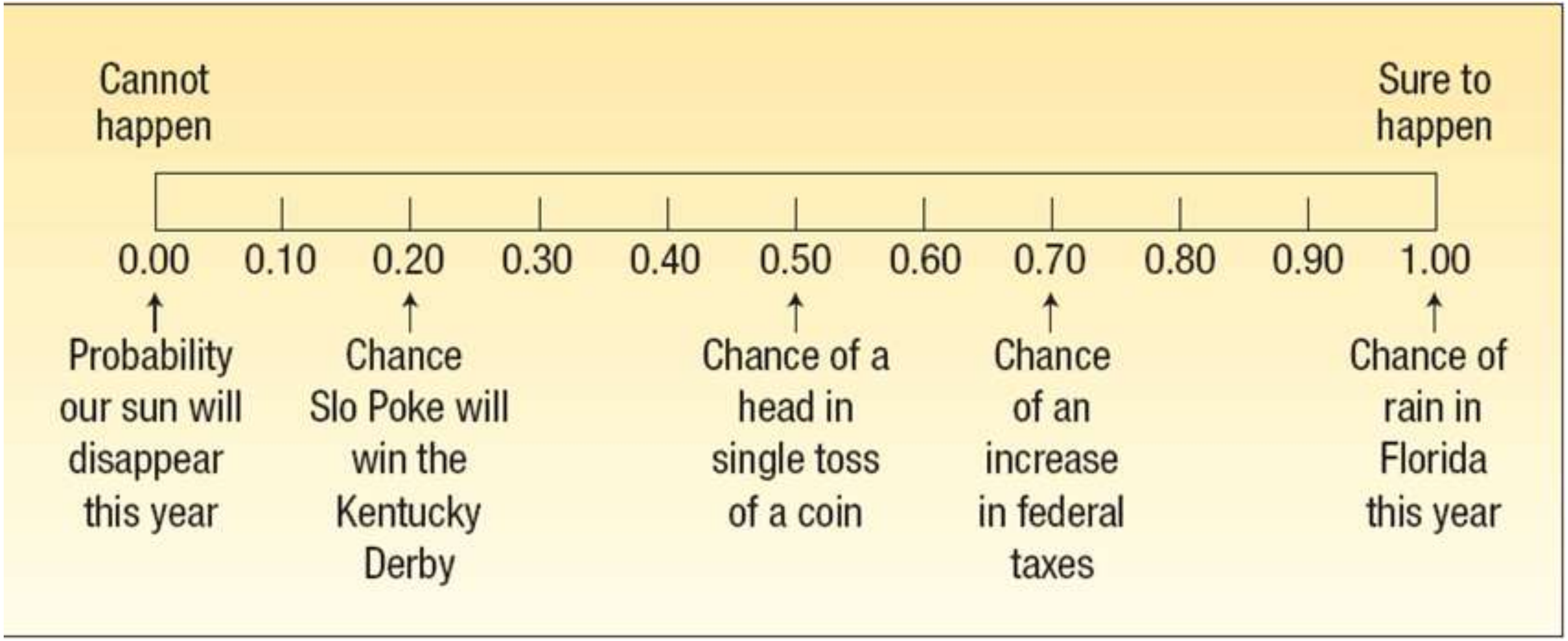
LO 5-6 Define the term *conditional probability.*

LO 5-7 Compute probabilities using a *contingency table*.

LO 5-8 Determine the number of outcomes using the appropriate principle of counting.

Probability

PROBABILITY A value between zero and one, inclusive, describing the relative possibility (chance or likelihood) an event will occur.



Experiment, Outcome, and Event

- An **experiment** is a process that leads to the occurrence of one, and only one, of several possible observations.
- An **outcome** is the particular result of an experiment.
- An **event** is the collection of one or more outcomes of an experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None are over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

LO 5-2 Identify and apply the appropriate approach to assigning probabilities.

Ways of Assigning Probability

Three ways of assigning probability:

1. **CLASSICAL PROBABILITY**

   Based on the assumption that the outcomes of an experiment are *equally likely.*

2. **EMPIRICAL PROBABILITY**

   The probability of an event happening is the fraction of the time similar events happened in the past.

3. **SUBJECTIVE CONCEPT OF PROBABILITY**

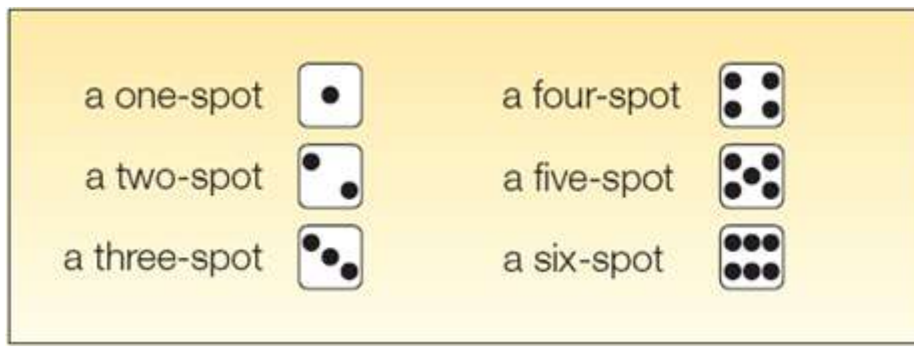
   The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

Classical Probability

CLASSICAL PROBABILITY $$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} \qquad [5\text{-}1]$$

Consider an experiment of rolling a six-sided die. What is the probability of the event that "an even number of spots appears face up"?

The possible outcomes are:



There are three "favorable" outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes.

Mutually Exclusive Events and Collectively Exhaustive Events

- Events are **mutually exclusive** if the occurrence of any one event means that none of the others can occur at the same time.
- Events are **independent** if the occurrence of one event does not affect the occurrence of another. Events are **collectively exhaustive** if at least one of the events must occur when an experiment is conducted.
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Empirical Probability

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

The empirical approach to probability is based on what is called the Law of Large Numbers.

Key to establishing probabilities empirically: a larger number of observations provides a more accurate estimate of the probability.

LAW OF LARGE NUMBERS Over a large number of trials, the empirical probability of an event will approach its true probability.

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Number of Trials	Number of Heads	Relative Frequency of Heads
1	0	.00
10	3	.30
50	26	.52
100	52	.52
500	236	.472
1,000	494	.494
10,000	5,027	.5027

Empirical Probability – Example

On February 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 123 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

$$\text{Probability of a successful flight} = \frac{\text{Number of successful flights}}{\text{Total number of flights}}$$

$$= \frac{121}{123} = 0.98$$

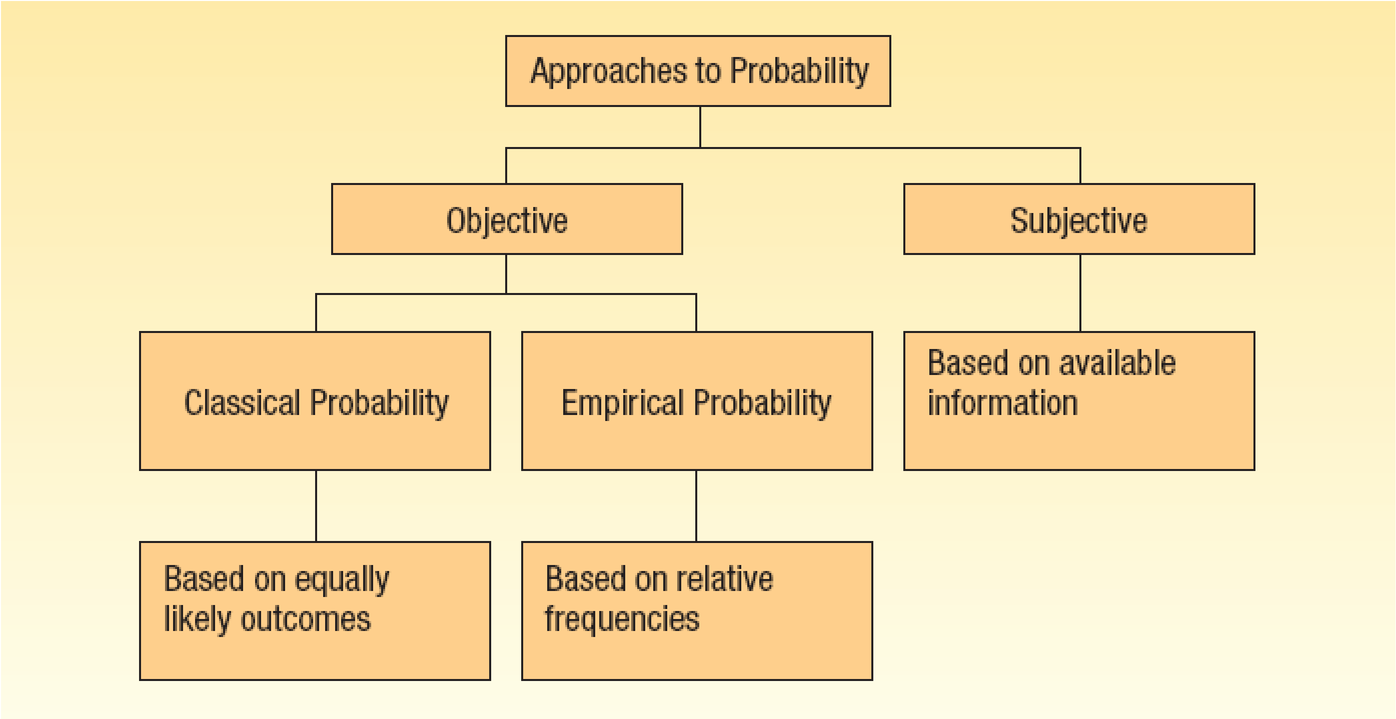
Subjective Probability – Example

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

- Use subjective probability if there is little or no past experience or information on which to base a probability.
- Illustrations of subjective probability are:

1. Estimating the likelihood the New England Patriots will play in the Super Bowl next year.
2. Estimating the likelihood you will be married before the age of 30.
3. Estimating the likelihood the U.S. budget deficit will be reduced by half in the next 10 years.

Summary of Types of Probability



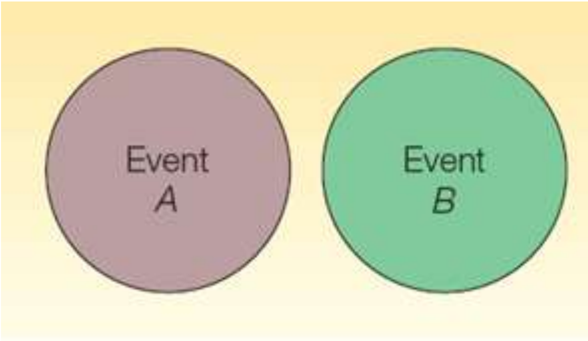
LO 5-3 Calculate probabilities using rules of addition.

Rules of Addition for Computing Probabilities

Rules of Addition

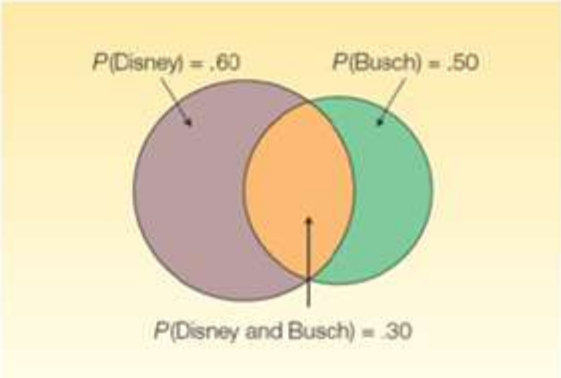
- Special Rule of Addition - If two events *A* and *B* are mutually exclusive, the probability of one *or* the other event's occurring equals the sum of their probabilities.

  P(A or B) = P(A) + P(B)



- The General Rule of Addition - If A and B are two events that are not mutually exclusive, then P(A or B) is given by the following formula:

  P(A or B) = P(A) + P(B) – P(A and B)



Addition Rule – Mutually Exclusive Events Example

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4,000 packages filled in the past month revealed:

Weight	Event	Number of Packages	Probability of Occurrence	
Underweight	*A*	100	.025	← $\frac{100}{4,000}$
Satisfactory	*B*	3,600	.900	
Overweight	*C*	300	.075	
		4,000	1.000	

What is the probability that a particular package will be either underweight or overweight?

P(A or C) = P(A) + P(C) = .025 + .075 = .10

Addition Rule – Not Mutually Exclusive Events Example

What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?

Card	Probability	Explanation
King	$P(A) = 4/52$	4 kings in a deck of 52 cards
Heart	$P(B) = 13/52$	13 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B) = 1/52$	1 king of hearts in a deck of 52 cards

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= 4/52 + 13/52 - 1/52$

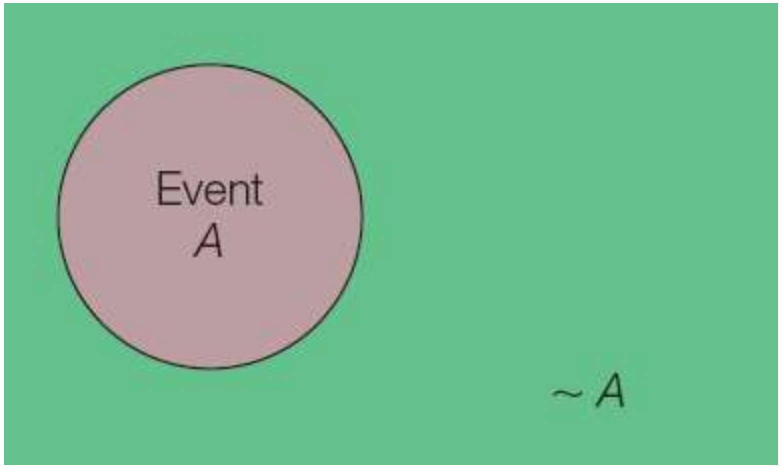
$= 16/52$, or .3077

The Complement Rule

The **complement rule** is used to determine the probability of an event occurring by subtracting the probability of the event *not* occurring from 1.

$$P(A) + P(\sim A) = 1$$

or $$P(A) = 1 - P(\sim A).$$

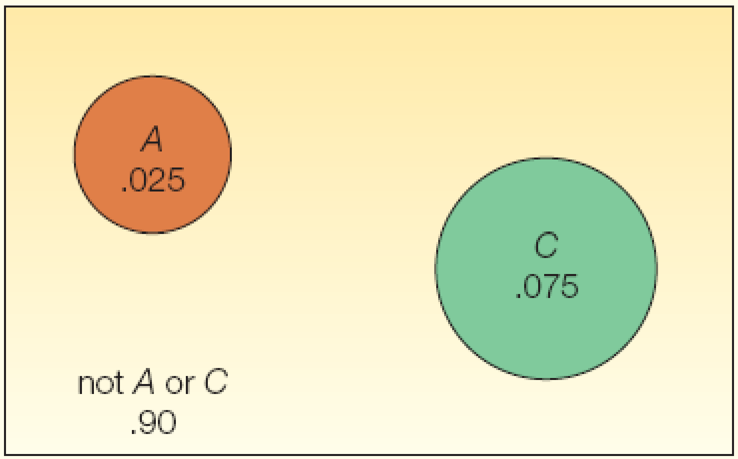


The Complement Rule – Example

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. Use the complement rule to show the probability of a satisfactory bag is 0.900.

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	*A*	100	.025
Satisfactory	*B*	3,600	.900
Overweight	*C*	300	.075
		4,000	1.000

The Complement Rule – Example



Weight	Event	Number of Packages	Probability of Occurrence
Underweight	*A*	100	.025
Satisfactory	*B*	3,600	.900
Overweight	*C*	300	.075
		4,000	1.000

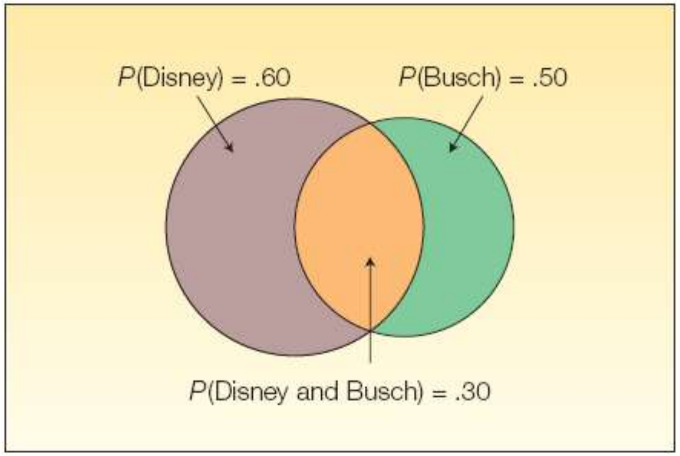
$$\begin{aligned} P(B) &= 1 - P(\sim B) \\ &= 1 - P(A \text{ or } C) \\ &= 1 - [P(A) + P(C)] \\ &= 1 - [.025 + .075] \\ &= 1 - .10 \\ &= .90 \end{aligned}$$

The General Rule of Addition

GENERAL RULE OF ADDITION $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **[5–4]**

The Venn Diagram shows the result of a survey of 200 tourists who visited Florida during the year. The survey revealed that 120 went to Disney World, 100 went to Busch Gardens, and 60 visited both.

What is the probability a selected person visited either Disney World or Busch Gardens?



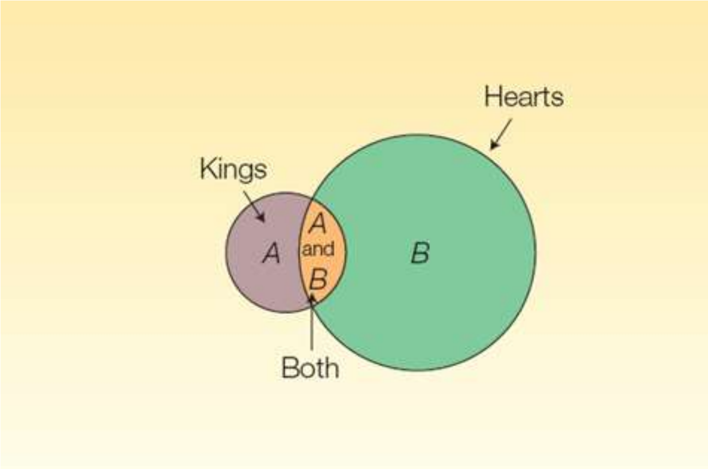
P(Disney or Busch) = P(Disney) + P(Busch) – P(both Disney and Busch)

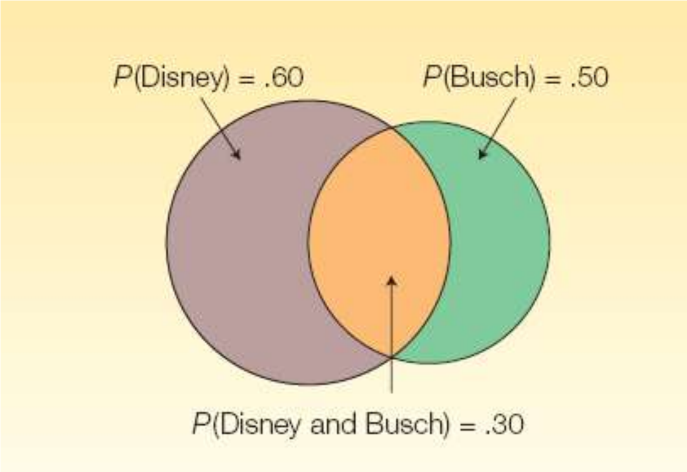
= 120/200 + 100/200 – 60/200

= .60 + .50 – .80

Joint Probability – Venn Diagram

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.

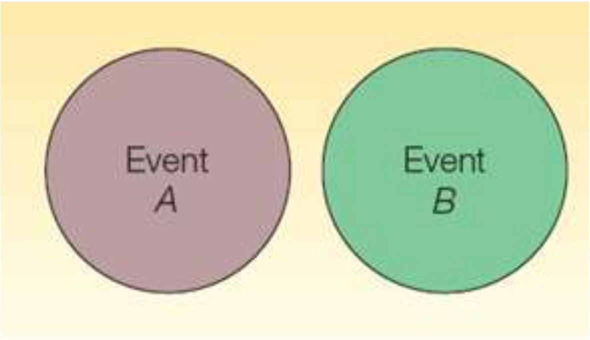




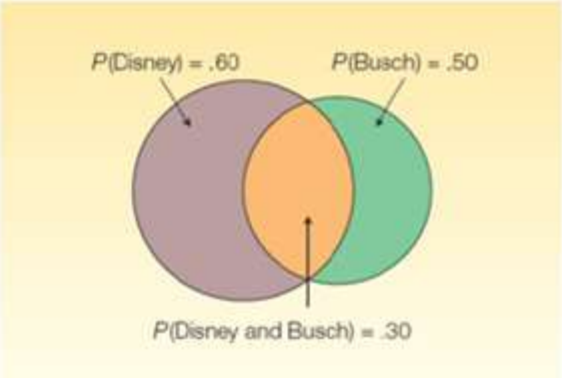
Joint Probability – Independent versus Dependent Events

Rules of Addition

- Independent Events
  P(A or B) = P(A) + P(B)



- Dependent Events
  P(A or B) = P(A) + P(B) – P(A and B)



LO 5-5 Calculate probabilities using the rules of multiplication.

Special Rule of Multiplication

- The **special rule of multiplication** requires that two events *A* and *B* are ***independent***.
- Two events *A* and *B* are independent if the occurrence of one has no effect on the probability of the occurrence of the other.
- This rule is written: $P(A \text{ and } B) = P(A)P(B)$

Multiplication Rule – Example

A survey by the American Automobile association (AAA) revealed 60 percent of its members made airline reservations last year. Two members are selected at random. Since the number of AAA members is very large, we can assume that R_1 *and* R_2 *are independent.*

What is the probability *both* made airline reservations last year?

Solution:

The probability the first member made an airline reservation last year is .60, written as $P(R_1) = .60$.

The probability that the second member selected made a reservation is also .60, so $P(R_2) = .60$.

Since the number of AAA members is very large, it can be assumed that R_1 and R_2 are independent.

$P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$

Conditional Probability

- A **conditional probability** is the probability of a particular event occurring, given that another event has occurred.

- The probability of the event *A* occurring given that the event *B* has occurred is written **P(A|B)**.

LO 5-6

General Multiplication Rule

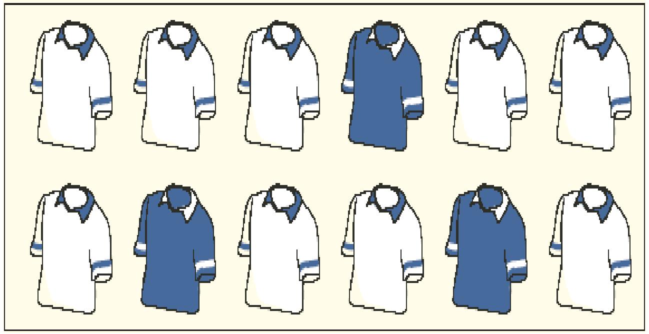
The **general rule of multiplication** is used to find the joint probability that two *independent* events will occur.

GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$ **[5-6]**

General Multiplication Rule – Example

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry.

What is the likelihood both shirts selected are white?



Probability Expression of the Question:

$P(W_1 \text{ and } W_2) = ?$

W_1 and W_2 are dependent

General Multiplication Rule – Example

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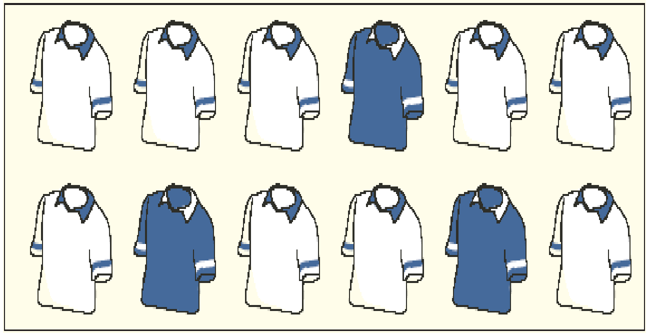


- First Day: $P(W_1) = 9/12$
- Second Day: $P(W_2 \mid W_1) = 8/11$.
- To determine the probability of selecting 2 white, we use formula:

  $P(AB) = P(A)\,P(B|A)$
- $P(W_1 \text{ and } W_2) = P(W_1)P(W_2 \mid W_1) = (9/12)(8/11) = 0.55$

LO 5-7 Compute probabilities using a contingency table.

Contingency Tables

A CONTINGENCY TABLE is a table used to classify sample observations according to two or more identifiable characteristics.

E.g., A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria—the **number of movies attended** and **gender**.

	Gender		
Movies Attended	**Men**	**Women**	**Total**
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

Contingency Tables – Example

A sample of executives were surveyed about their loyalty to their company. One of the questions was, "If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?" The responses of the 200 executives in the survey were cross-classified with their length of service with the company.

Loyalty	Length of Service: Less than 1 Year, B_1	1–5 Years, B_2	6–10 Years, B_3	More than 10 Years, B_4	Total
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

Contingency Tables – Example

Event A_1 - if a randomly selected executive will remain with the company despite an equal or slightly better offer from another company. Since there are 120 executives out of the 200 in the survey who would remain with the company

$P(A_1)$ = 120/200, or .60

Event B_4 - if a randomly selected executive has more than 10 years of service with the company. Thus, $P(B_4 | A_1)$ is the conditional probability that an executive with more than 10 years of service would remain with the company. Of the 120 executives who would remain, 75 have more than 10 years of service, so

$P(B_4 | A_1)$ = 75/120

$$P(A_1 \text{ and } B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{9{,}000}{24{,}000} = .375$$

LO 5-8 Determine the number of outcomes using the appropriate principle of counting.

Counting Rules – Multiplication

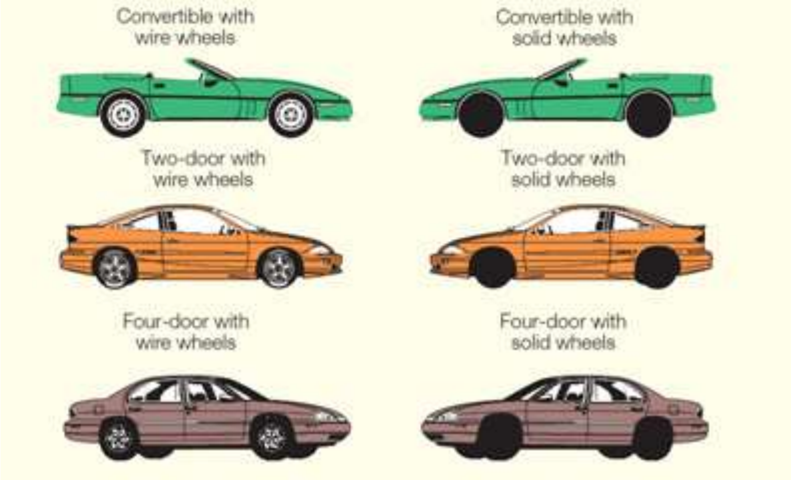
The **multiplication formula** indicates that if there are *m* ways of doing one thing and *n* ways of doing another thing, there are *m* x *n* ways of doing both.

Example: Dr. Delong has 10 shirts and 8 ties. How many shirt and tie outfits does he have?

(10)(8) = 80

Counting Rules – Multiplication: Example

An automobile dealer wants to advertise that for $29,999 you can buy a convertible, a two-door sedan, or a four-door model with your choice of either wire wheel covers or solid wheel covers. How many different arrangements of models and wheel covers can the dealer offer?



MULTIPLICATION FORMULA Total number of arrangements = $(m)(n)$ **[5–8]**

We can employ the multiplication formula as a check (where m is the number of models and n the wheel cover type). From formula (5–8):

$$\text{Total possible arrangements} = (m)(n) = (3)(2) = 6$$

Counting Rules – Permutation

A **permutation** is any arrangement of r objects selected from n possible objects. The order of arrangement is important in permutations.

PERMUTATION FORMULA

$$_nP_r = \frac{n!}{(n-r)!} \quad [5–9]$$

where:

n is the total number of objects.

r is the number of objects selected.

Permutation Example

A group of three electronic parts are to be assembled in any order. How many different ways can they be assembled?

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_3P_3 = \frac{3!}{(3-3)!} = 6 $$

Permutation – Another Example

Betts Machine Shop Inc. has eight screw machines but only three spaces available in the production area for the machines. In how many different ways can the eight machines be arranged in the three spaces available?

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

$$ {}_8P_3 = \frac{8!}{(8-3)!} = 336 $$

Counting Rules – Combination

A **combination** is the number of ways to choose *r* objects from a group of *n* objects without regard to order.

COMBINATION FORMULA

$$ {}_nC_r = \frac{n!}{r!(n-r)!} \quad [5\text{–}10] $$

where:

n is the total number of objects.

r is the number of objects selected.

Combination Example

The marketing department has been given the assignment of designing color codes for the 42 different lines of compact disks sold by Goody Records. Three colors are to be used on each CD, but a combination of three colors used for one CD cannot be rearranged and used to identify a different CD. This means that if green, yellow, and violet were used to identify one line, then yellow, green, and violet (or any other combination of these three colors) cannot be used to identify another line. Would seven colors taken three at a time be adequate to color-code the 42 lines?

Combination Example

Would 7 colors taken 3 at a time be adequate to color-code the 42 lines?

$n = 7$ colors to choose from

$r = 3$ colors to choose each time

Order of color selection is not important

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_7C_3 = \frac{7!}{3!(7-3)!} = 35$$

Combination – Another Example

There are 12 players on the Carolina Forest High School basketball team. Coach Thompson must pick 5 players among the 12 on the team to comprise the starting lineup. How many different groups are possible?

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

$$ {}_{12}C_5 = \frac{12!}{5!(12-5)!} = 792 $$