Describing Data:
Numerical Measures

Chapter 03

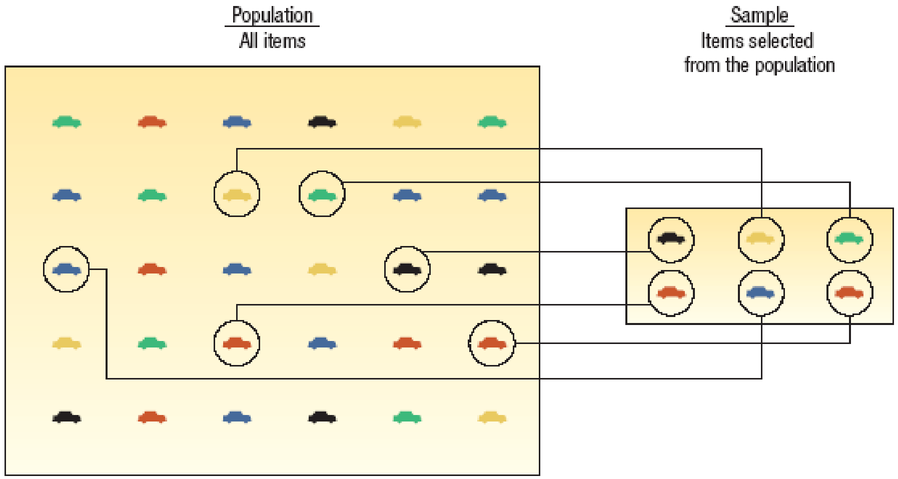
LEARNING OBJECTIVES

LO 3-1 Explain the concept of central tendency.
LO 3-2 Identify and compute the arithmetic mean.
LO 3-3 Compute and interpret the weighted mean.
LO 3-4 Determine the median.
LO 3-5 Identify the mode.
LO 3-6 Explain and apply measures of dispersion.
LO 3-7 Compute and explain the variance and the standard deviation.
LO 3-8 Explain Chebyshev’s Theorem and the Empirical Rule.

Parameter vs. Statistics

PARAMETER A measurable characteristic of a *population*.

STATISTIC A measurable characteristic of a *sample*.



LO 3-1 Explain the concept of central tendency.

Central Tendency – Measures of Location

- The purpose of a measure of location is to pinpoint the center of a distribution of data.
- There are many measures of location. We will consider four:
    1. The arithmetic mean
    2. The weighted mean
    3. The median
    4. The mode

LO 3-2 Identify and compute the arithmetic mean.

Characteristics of the Mean

- The arithmetic mean is the most widely used measure of location.
- Requires the interval scale.
- Major characteristics:
  - All values are used.
  - It is unique.
  - The sum of the deviations from the mean is 0.
  - It is calculated by summing the values and dividing by the number of values.

LO 3-2

Population Mean

For ungrouped data, the population mean is the sum of all the population values divided by the total number of population values:

POPULATION MEAN $$\mu = \frac{\Sigma X}{N} \qquad [3\text{–}1]$$

where:

- μ represents the population mean. It is the Greek lowercase letter "mu."
- N is the number of values in the population.
- X represents any particular value.
- Σ is the Greek capital letter "sigma" and indicates the operation of adding.
- ΣX is the sum of the X values in the population.

EXAMPLE – Population Mean

There are 42 exits on I-75 through the state of Kentucky. Listed below are the distances between exits (in miles).

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Why is this information a population?
What is the mean number of miles between exits?

EXAMPLE – Population Mean

There are 42 exits on I-75 through the state of Kentucky. Listed below are the distances between exits (in miles).

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Why is this information a population?

This is a population because we are considering all the exits on I-75 in the state of Kentucky.

What is the mean number of miles between exits?

$$\mu = \frac{\Sigma X}{N} = \frac{11 + 4 + 10 + \cdots + 1}{42} = \frac{192}{42} = 4.57$$

Sample Mean

For ungrouped data, the sample mean is the sum of all the sample values divided by the number of sample values:

SAMPLE MEAN

$$\bar{X} = \frac{\Sigma X}{n} \qquad [3\text{–}2]$$

where:

$\bar{X}$ is the sample mean. It is read "X bar."

n is the number of values in the sample.

EXAMPLE – Sample Mean

SunCom is studying the number of minutes used monthly by clients in a particular cell phone rate plan. A random sample of 12 clients showed the following number of minutes used last month.

90	77	94	89	119	112
91	110	92	100	113	83

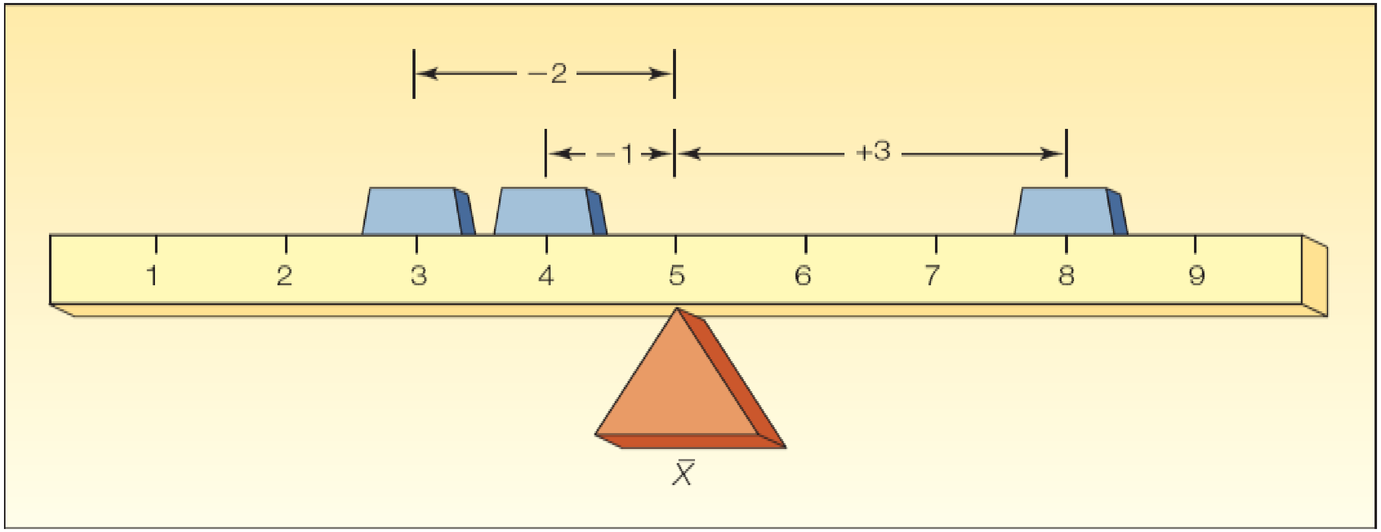
What is the arithmetic mean number of minutes used?

$$\text{Sample mean} = \frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$$

$$\overline{X} = \frac{\Sigma X}{n} = \frac{90 + 77 + \cdots + 83}{12} = \frac{1170}{12} = 97.5$$

Properties of the Arithmetic Mean

1. Every set of interval-level and ratio-level data has a mean.
2. All the values are included in computing the mean.
3. The mean is unique.
4. The sum of the deviations of each value from the mean is zero.



LO 3-3 Compute and interpret the weighted mean.

Weighted Mean

The weighted mean of a set of numbers X_1, X_2, ..., X_n, with corresponding weights w_1, w_2, ...,w_n, is computed from the following formula:

WEIGHTED MEAN

$$\bar{X}_w = \frac{w_1X_1 + w_2X_2 + w_3X_3 + \cdots + w_nX_n}{w_1 + w_2 + w_3 + \cdots + w_n} \quad [3\text{–}3]$$

EXAMPLE – Weighted Mean

The Carter Construction Company pays its hourly employees \$16.50, \$19.00, or \$25.00 per hour. There are 26 hourly employees, 14 of whom are paid at the \$16.50 rate, 10 at the \$19.00 rate, and 2 at the \$25.00 rate.

What is the mean hourly rate paid the 26 employees?

$$\bar{X}_w = \frac{14(\$16.50) + 10(\$19.00) + 2(\$25.00)}{14 + 10 + 2} = \frac{\$471.00}{26} = \$18.1154$$

The Median

MEDIAN The midpoint of the values after they have been ordered from the smallest to the largest, or the largest to the smallest.

PROPERTIES OF THE MEDIAN

1. There is a unique median for each data set.
2. It is not affected by extremely large or small values and is therefore a valuable measure of central tendency when such values occur.
3. It can be computed for ratio-level, interval-level, and ordinal-level data.
4. It can be computed for an open-ended frequency distribution if the median does not lie in an open-ended class.

EXAMPLES – Median

The ages for a sample of five college students are:

21, 25, 19, 20, 22

Arranging the data in ascending order gives:

19, 20, 21, 22, 25.

Thus the median is 21.

The heights of four basketball players, in inches, are:

76, 73, 80, 75

Arranging the data in ascending order gives:

73, 75, 76, 80.

Thus the median is 75.5

EXAMPLES – Median

Facebook is a popular social networking website. Users can add friends and send them messages, and update their personal profiles to notify friends about themselves and their activities. A sample of 10 adults revealed they spent the following number of hours last month using Facebook. Find the median number of hours.

3	5	7	5	9	1	3	9	17	10

EXAMPLES – Median

Original sample data set:

3	5	7	5	9	1	3	9	17	10

Step 1: Sort-order the data

1	3	3	5	5	7	9	9	10	17

Step 2: Find the median

$$Median = \frac{5+7}{2} = 6$$

The Mode

MODE The value of the observation that appears most frequently.

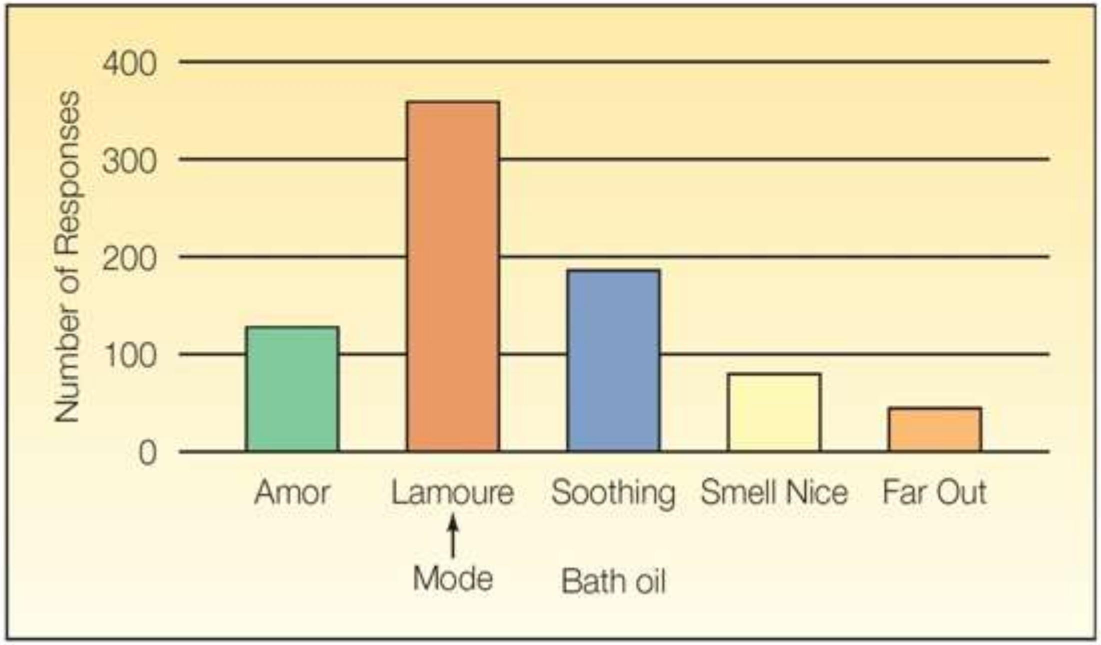


CHART 3–1 Number of Respondents Favoring Various Bath Oils

LO 3-5

Example – Mode

Using the data regarding the distance in miles between exits on I-75 through Kentucky. The information is repeated below. What is the modal distance?

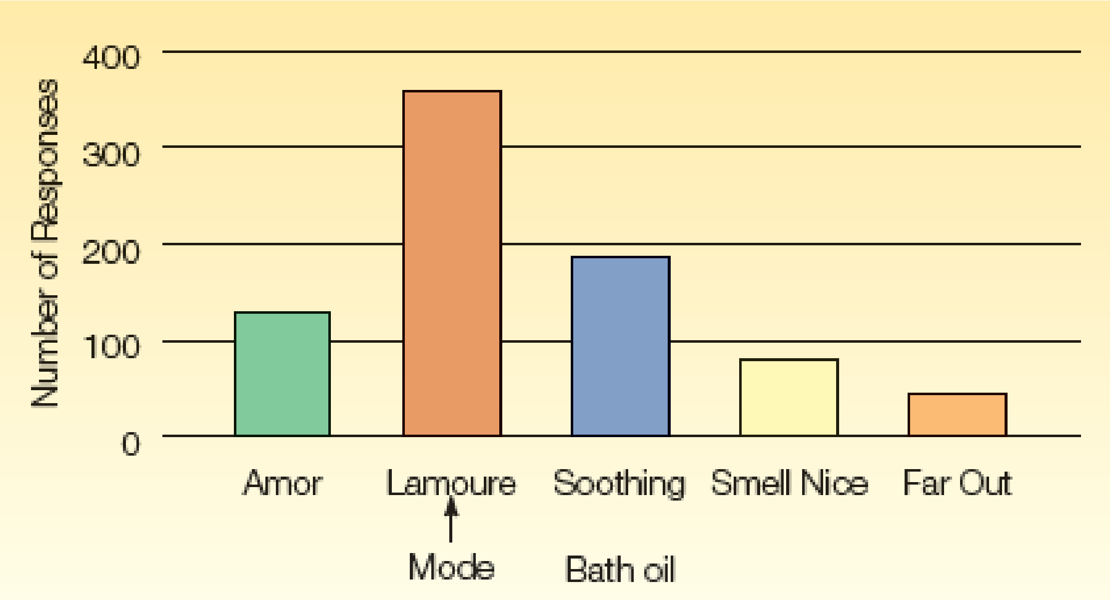
Organize the distances into a frequency table.

11	4	10	4	9	3	8	10	3	14	1	10	3	5
2	2	5	6	1	2	2	3	7	1	3	7	8	10
1	4	7	5	2	2	5	1	1	3	3	1	2	1

Distance in Miles between Exits	Frequency
1	8
2	7
3	7
4	3
5	4
6	1
7	3
8	2
9	1
10	4
11	1
14	1
Total	42

Example – Mode for Nominal Data

A company has developed five bath oils. The bar chart on the right shows the results of a marketing survey designed to find which bath oil consumers prefer.



Mean, Median, Mode Using Excel

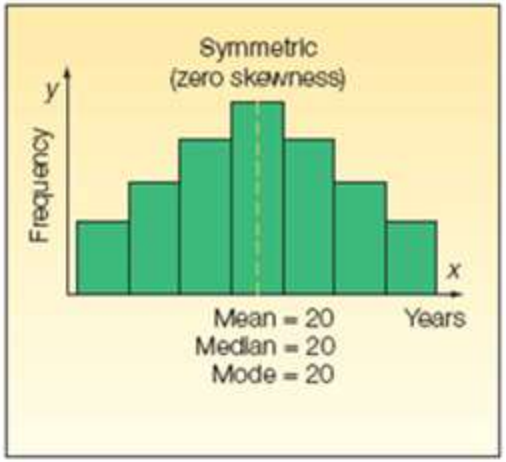
Table 2–4 in Chapter 2 showed the profits of the 80 vehicles sold last month at Whitner Autoplex in Raytown, Missouri. Determine the mean and the median selling price. The mean and the median selling prices are reported in the following Excel output. There are 80 vehicles in the study. So the calculations with a calculator would be tedious and prone to error.

APPLEWOOD AUTO GROUP 2011

	A	B	C	D	E	F	G	H
1	Age	Profit	Location	Vehicle-Type	Previous		*Profit*	
2	33	$1,889	Olean	SUV	1			
3	47	$1,461	Kane	Sedan	0		Mean	1843.17
4	44	$1,532	Tionesta	SUV	3		Standard Error	47.97
5	53	$1,220	Olean	Sedan	0		Median	1882.50
6	51	$1,674	Sheffield	Sedan	1		Mode	1915.00
7	41	$2,389	Kane	Truck	1		Standard Deviation	643.63
8	58	$2,058	Kane	SUV	1		Sample Variance	414256.61
9	35	$1,919	Tionesta	SUV	1		Kurtosis	-0.22
10	45	$1,266	Olean	Sedan	0		Skewness	-0.24
11	54	$2,991	Tionesta	Sedan	0		Range	2998
12	56	$2,695	Kane	Sedan	2		Minimum	294
13	41	$2,165	Tionesta	SUV	0		Maximum	3292
14	38	$1,766	Sheffield	SUV	0		Sum	331770
15	48	$1,952	Tionesta	Compact	1		Count	180

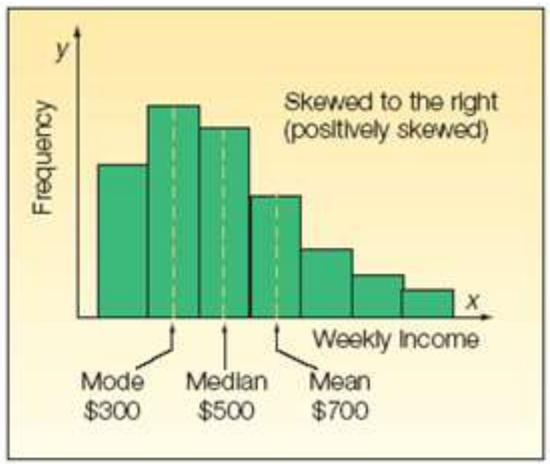
LO 3-2,4,5

The Relative Positions of the Mean, Median, and the Mode



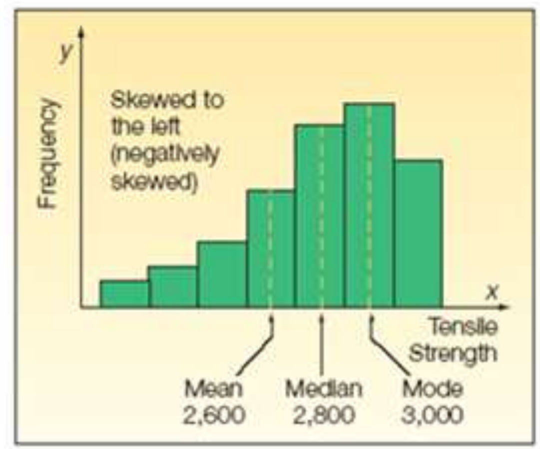
zero skewness

mode = median = mean



positive skewness

mode < median < mean



negative skewness

mode > median > mean

Dispersion

LO 3-6 Explain and apply measures of dispersion.

A measure of location, such as the mean or the median, only describes the center of the data. It is valuable from that standpoint, but it does not tell us anything about the *spread* of the data.

For example, if your nature guide told you that the river ahead averaged 3 feet in depth, would you want to wade across on foot without additional information? Probably not. You would want to know something about the variation in the depth.

A second reason for studying the dispersion in a set of data is to compare the spread in two or more distributions.

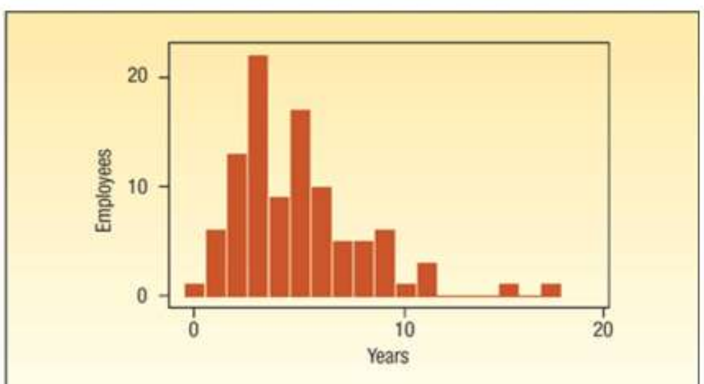


CHART 3–5 Histogram of Years of Employment at Hammond Iron Works, Inc.

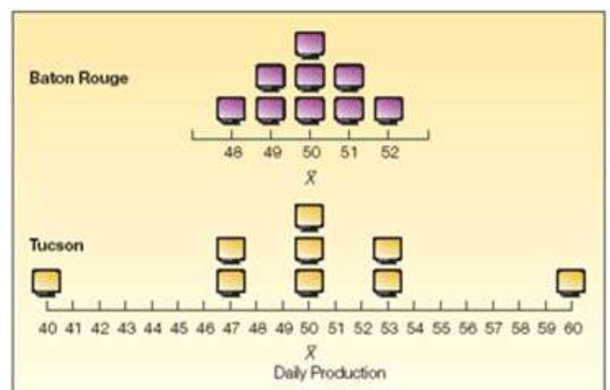


CHART 3–6 Hourly Production of Computer Monitors at the Baton Rouge and Tucson Plants

Measures of Dispersion

- Range

RANGE Range = Largest value − Smallest value [3–6]

- Mean Deviation

MEAN DEVIATION
$$MD = \frac{\Sigma|X - \overline{X}|}{n} \quad [3\text{–}7]$$

- Variance and Standard Deviation

POPULATION VARIANCE
$$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N} \quad [3\text{–}8]$$

POPULATION STANDARD DEVIATION
$$\sigma = \sqrt{\frac{\Sigma(X - \mu)^2}{N}} \quad [3\text{–}9]$$

EXAMPLE – Range

The number of cappuccinos sold at the Starbucks location in the Orange Country Airport between 4 and 7 p.m. for a sample of 5 days last year were 20, 40, 50, 60, and 80. Determine the range for the number of cappuccinos sold.

$$\text{Range} = \text{Largest} - \text{Smallest value}$$
$$= 80 - 20 = 60$$

Mean Deviation

MEAN DEVIATION The arithmetic mean of the absolute values of the deviations from the arithmetic mean.

- A shortcoming of the range is that it is based on only two values, the highest and the lowest; it does not take into consideration all of the values.
- The **mean deviation** does. It measures the mean amount by which the values in a population, or sample, vary from their mean

MEAN DEVIATION

$$MD = \frac{\Sigma|X - \bar{X}|}{n} \qquad [3\text{–}7]$$

where:

X is the value of each observation.
$\bar{X}$ is the arithmetic mean of the values.
n is the number of observations in the sample.
$||$ indicates the absolute value.

EXAMPLE – Mean Deviation

The number of cappuccinos sold at the Starbucks location in the Orange Country Airport between 4 and 7 p.m. for a sample of 5 days last year were 20, 40, 50, 60, and 80.

Determine the mean deviation for the number of cappuccinos sold.

Step 1: Compute the mean

$$\bar{x} = \frac{\sum x}{n} = \frac{20+40+50+60+80}{5} = 50$$

EXAMPLE – Mean Deviation

Step 2: Subtract the mean (50) from each of the observations, convert to positive if difference is negative

Step 3: Sum the absolute differences found in step 2, then divide by the number of observations

Number of Cappuccinos Sold Daily	$(X - \overline{X})$	Absolute Deviation
20	$(20 - 50) = -30$	30
40	$(40 - 50) = -10$	10
50	$(50 - 50) = 0$	0
60	$(60 - 50) = 10$	10
80	$(80 - 50) = 30$	30
	Total	80

$$MD = \frac{\Sigma|X - \overline{X}|}{n} = \frac{80}{5} = 16$$

LO 3-7 Compute and interpret the standard deviation.

Variance and Standard Deviation

VARIANCE The arithmetic mean of the squared deviations from the mean.

STANDARD DEVIATION The square root of the variance.

- The variance and standard deviations are nonnegative and are zero only if all observations are the same.
- For populations whose values are *near the mean*, the variance and standard deviation will be small.
- For populations whose values are *dispersed from the mean*, the population variance and standard deviation will be large.
- The variance overcomes the weakness of the range by using all the values in the population

Variance – Formula and Computation

POPULATION VARIANCE $$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N} \qquad [3\text{–}8]$$

σ^2 is the population variance (σ is the lowercase Greek letter sigma). It is read as "sigma squared."
X is the value of an observation in the population.
μ is the arithmetic mean of the population.
N is the number of observations in the population.

Steps in Computing the Variance.

Step 1: Find the mean.
Step 2: Find the difference between each observation and the mean, and square that difference.
Step 3: Sum all the squared differences found in step 2.
Step 4: Divide the sum of the squared differences by the number of items in the population.

EXAMPLE – Variance and Standard Deviation

The number of traffic citations issued during the last five months in Beaufort County, South Carolina, is reported below:

Month	January	February	March	April	May	June	July	August	September	October	November	December
Citations	19	17	22	18	28	34	45	39	38	44	34	10

What is the population variance?

Step 1: Find the mean.

$$\mu = \frac{\sum x}{N} = \frac{19+17+\ldots+34+10}{12} = \frac{348}{12} = 29$$

EXAMPLE – Variance and Standard Deviation

The number of traffic citations issued during the last five months in Beaufort County, South Carolina, is reported below:

Month	January	February	March	April	May	June	July	August	September	October	November	December
Citations	19	17	22	18	28	34	45	39	38	44	34	10

What is the population variance?

Step 2: Find the difference between each observation and the mean, and square that difference.

Step 3: Sum all the squared differences found in step 2.

Step 4: Divide the sum of the squared differences by the number of items in the population.

EXAMPLE – Variance and Standard Deviation

The number of traffic citations issued during the last 12 months in Beaufort County, South Carolina, is reported below:

Month	January	February	March	April	May	June	July	August	September	October	November	December
Citations	19	17	22	18	28	34	45	39	38	44	34	10

What is the population variance?

Step 2: Find the difference between each observation and the mean, and square that difference.

Step 3: Sum all the squared differences found in step 3.

Step 4: Divide the sum of the squared differences by the number of items in the population.

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{1{,}488}{12} = 124$$

Month	Citations (X)	$X - \mu$	$(X - \mu)^2$
January	19	−10	100
February	17	−12	144
March	22	−7	49
April	18	−11	121
May	28	−1	1
June	34	5	25
July	45	16	256
August	39	10	100
September	38	9	81
October	44	15	225
November	34	5	25
December	10	−19	361
Total	348	0	1,488

Sample Variance

SAMPLE VARIANCE $$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1}$$ **[3–10]**

Where :

s^2 is the sample variance

X is the value of each observation in the sample

$\overline{X}$ is the mean of the sample

n is the number of observations in the sample

EXAMPLE – Sample Variance

The hourly wages for a sample of part-time employees at Home Depot are: $12, $20, $16, $18, and $19.

What is the sample variance?

SAMPLE VARIANCE $$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1}$$ [3–10]

Hourly Wage (X)	$X - \overline{X}$	$(X - \overline{X})^2$
$12	−$5	25
20	3	9
16	−1	1
18	1	1
19	2	4
$85	0	40

$$s^2 = \frac{\Sigma(X - \overline{X})^2}{n - 1} = \frac{40}{5 - 1}$$

$= 10$ in dollars squared

LO 3-7

Sample Standard Deviation

SAMPLE STANDARD DEVIATION $$s = \sqrt{\frac{\Sigma(X - \overline{X})^2}{n - 1}}$$ **[3–11]**

Where :

s^2 is the sample variance

X is the value of each observation in the sample

$\overline{X}$ is the mean of the sample

n is the number of observations in the sample

LO 3-7

Interpretations and Uses of the Standard Deviation

The standard deviation is commonly used as a measure to compare the spread in two or more sets of observations.

	Biweekly $ Invested in Dupree Paint Company	
	Georgia	Texas
Standard Deviation	$7.51	$10.74

LO 3-8 **LO 3-8** Explain Chebyshev's Theorem and the Empirical Rule.

Chebyshev's Theorem

The arithmetic mean biweekly amount contributed by the Dupree Paint employees to the company's profit-sharing plan is \$51.54, and the standard deviation is \$7.51. At least what percent of the contributions lie within plus 3.5 standard deviations and minus 3.5 standard deviations of the mean?

CHEBYSHEV'S THEOREM For any set of observations (sample or population), the proportion of the values that lie within k standard deviations of the mean is at least $1 - 1/k^2$, where k is any constant greater than 1.

$$1 - \frac{1}{k^2} = 1 - \frac{1}{(3.5)^2} = 1 - \frac{1}{12.25} = 0.92$$

The Empirical Rule

EMPIRICAL RULE For a symmetrical, bell-shaped frequency distribution, approximately 68 percent of the observations will lie within plus and minus one standard deviation of the mean; about 95 percent of the observations will lie within plus and minus two standard deviations of the mean; and practically all (99.7 percent) will lie within plus and minus three standard deviations of the mean.

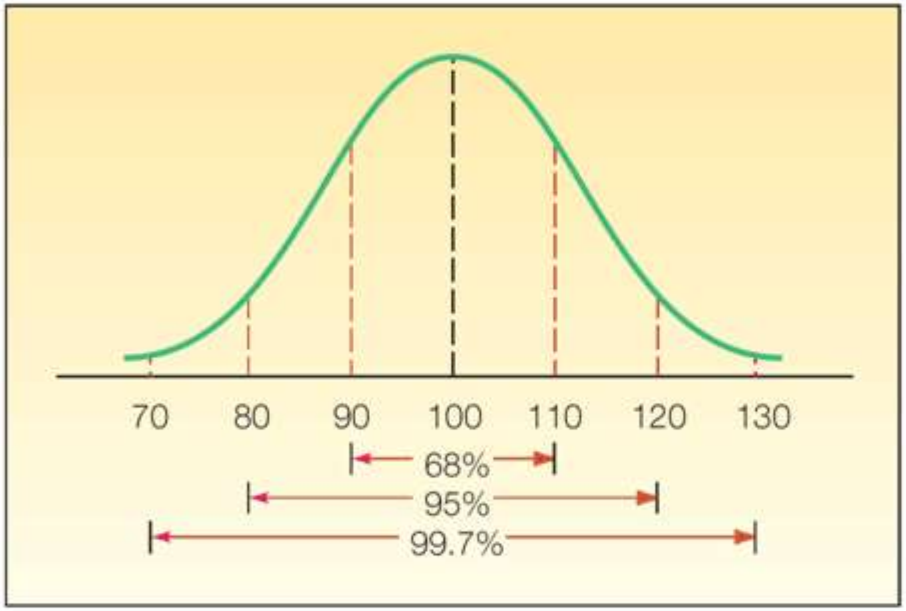


CHART 3–7 A Symmetrical, Bell-Shaped Curve Showing the Relationships between the Standard Deviation and the Observations

Ethics and Reporting Rules

Ethical and unbiased reporting of statistical results requires:

(1) Learning about how to organize, summarize, and interpret data using statistics, and
(2) Understanding statistics so that
(3) You can be an intelligent consumer and provider of information.